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Large deformations of the Tracy-Widom distribution. I: Non-oscillatory asymptotics. (English) Zbl 1407.60005

Commun. Math. Phys. 359, No. 1, 223-263 (2018).

Summary: We analyze the left-tail asymptotics of deformed Tracy-Widom distribution functions describing the fluctuations of the largest eigenvalue in invariant random matrix ensembles after removing each soft edge eigenvalue independently with probability $1-\gamma \in [0,1]$. As γ varies, a transition from Tracy-Widom statistics ($\gamma = 1$) to classical Weibull statistics ($\gamma = 0$) was observed in the physics literature by *O. Bohigas* et al. ["Deformations of the Tracy-Widom distribution", Phys. Rev. E 79, No. 3, Article ID 031117, 6 p. (2009; doi:10.1103/physreve.79.031117)]. We provide a description of this transition by rigorously computing the leading-order left-tail asymptotics of the thinned GOE, GUE, and GSE Tracy-Widom distributions. In this paper, we obtain the asymptotic behavior in the non-oscillatory region with $\gamma \in [0,1)$ fixed (for the GOE, GUE, and GSE distributions) and $\gamma \uparrow 1$ at a controlled rate (for the GUE distribution). This is the first step in an ongoing program to completely describe the transition between Tracy-Widom and Weibull statistics. As a corollary to our results, we obtain a new total-integral formula involving the Ablowitz-Segur solution to the second Painlevé equation.

MSC:

60B20 Random matrices (probabilistic aspects)
15B52 Random matrices (algebraic aspects)
60F10 Large deviations

Keywords:

Tracy-Widom distribution functions; Ablowitz-Segur solution; Painlevé equation

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