Darrall, Bradley T.; **Hadjesfandiari, Ali R.**; **Dargush, Gary F.**

Size-dependent piezoelectricity: a 2D finite element formulation for electric field-mean curvature coupling in dielectrics. (English) Zbl 1406.74213

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Summary: The classical theory of piezoelectricity defines linear size-independent electromechanical response in non-centrosymmetric dielectrics that involves coupling between the electric field and the mechanical strains. However, with the continuing push to develop novel micro- and nano-scale materials, structures and devices, there is a need to refine and explore size-dependent electro-mechanical coupling phenomena, which have been observed in experiments on centrosymmetric dielectrics. Here a finite element variational formulation is developed based upon a recent consistent size-dependent theory that incorporates the interactions between the electric field and the mechanical mean curvatures in dielectrics, including those with centrosymmetric structure. The underlying formulation is theoretically consistent in several important aspects. In particular, the electric field equations are consistent with Maxwell's equations, while the mechanical field equations are based upon the recent consistent couple stress theory, involving skew-symmetric mean curvature and couple stress tensors. This, in turn, permits the development of a fully-consistent finite element method for the solution of size-dependent piezoelectric boundary value problems. In this paper, an overview of size-dependent piezoelectricity is first provided, followed by the development of the variational formulation and finite element representation specialized for the planar response of centrosymmetric cubic and isotropic materials. The new formulation is then applied to several illustrative examples to bring out important characteristics predicted by this consistent size-dependent piezoelectric theory.

MSC:

74F15 Electromagnetic effects in solid mechanics
74S05 Finite element methods applied to problems in solid mechanics

Keywords:

flexoelectricity; couple stresses; size-dependent multiphysics

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