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One-dimensional constitutive SMA model with two martensite variants: analytical and numerical solutions. (English) Zbl 1406.74031

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Summary: This paper deals with the one-dimensional modeling of a shape memory alloy (SMA) in order to reproduce the special thermo-mechanical response of the material. In particular, new analytical solutions are developed using a simple one-dimensional SMA model based on the introduction of three phases: tensile martensite, compressive martensite and austenite; the ability to reproduce the pseudo-elastic and shape memory effects; the different behavior in tension and compression; the different elastic properties of the three phases; and the reorientation process of the martensite. The model assumes the martensite volume fractions as internal variables, whose evolution is governed by stress and temperature. Analytical solutions, which take into consideration the axial loads and thermal processes, are developed to solve the axial problem. Furthermore, a numerical procedure is developed in order to time-integrate the kinetic laws that rule the evolution of the internal variables during the phase transformations and a 2-node finite element beam is implemented. Considering the constant and non-constant elastic properties for the three SMA phases, some applications are presented in order to verify the effectiveness of the proposed model and the analytical and numerical solutions. Comparisons with experimental data related to the axial response of SMA wires and laminae are carried out. Finally, a numerical application concerning a SMA lamina, which is subjected to bending load at high temperatures inducing the pseudo-elastic effect in the material, is presented.

MSC:

74A20 Theory of constitutive functions in solid mechanics
74H30 Regularity of solutions of dynamical problems in solid mechanics
74D05 Linear constitutive equations for materials with memory
74M05 Control, switches and devices (“smart materials”) in solid mechanics

Keywords:

one-dimensional SMA model; two martensite variants; non-constant elastic properties; tension-compression asymmetry; pseudo-elastic and shape memory effects; analytical and numerical solutions

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