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Effective potentials in nonlinear polycrystals and quadrature formulae. (English)

Zbl 1404.74143

Proc. R. Soc. Lond., A, Math. Phys. Eng. Sci. 473, No. 2204, Article ID 20170213, 20 p. (2017).

Summary: This study presents a family of estimates for effective potentials in nonlinear polycrystals. Noting that these potentials are given as averages, several quadrature formulae are investigated to express these integrals of nonlinear functions of local fields in terms of the moments of these fields. Two of these quadrature formulae reduce to known schemes, including a recent proposition [*P. Ponte Castañeda*, Proc. R. Soc. Lond., A, Math. Phys. Eng. Sci. 471, No. 2184, Article ID 20150665, 20 p. (2015; Zbl 1371.74057)] obtained by completely different means. Other formulae are also reviewed that make use of statistical information on the fields beyond their first and second moments. These quadrature formulae are applied to the estimation of effective potentials in polycrystals governed by two potentials, by means of a reduced-order model proposed by the authors (non-uniform transformation field analysis). It is shown how the quadrature formulae improve on the tangent second-order approximation in porous crystals at high stress triaxiality. It is found that, in order to retrieve a satisfactory accuracy for highly nonlinear porous crystals under high stress triaxiality, a quadrature formula of higher order is required.

MSC:

74Q15 Effective constitutive equations in solid mechanics
74S30 Other numerical methods in solid mechanics (MSC2010)
74N05 Crystals in solids
65D30 Numerical integration

Keywords:

homogenization; model reduction; numerical integration; field statistics; crystal plasticity

Full Text: DOI

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