

**Jeřábek, Emil**

**Galois connection for multiple-output operations.** (English) Zbl 1402.08002  
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The paper presents a generalization of a special type of Galois connection, namely, given a base set  $B$ , instead of classes of operations  $f : B^n \rightarrow B$ , the partial, multi-valued functions  $f : B^n \rightarrow B^m$ ,  $m \geq 0$  are considered. Functions  $f : B^k \rightarrow M$  valued in partially ordered monoids  $M$  as invariants are also used. As main results the paper presents:

- (1) the main Galois connection between partial multi-valued multi-output functions and pomonoid-valued weight functions in its most general form;
- (2) variants of Galois connections for restricted classes of multi-output functions or weights;
- (3) Galois connections for classes of total multi-output functions;
- (4) a description of finitely generated subdirectly irreducible commutative monoids.

Reviewer: [Lavinia Ciungu \(Iowa City\)](#)

**MSC:**

- 08A40** Operations and polynomials in algebraic structures, primal algebras
- 06F05** Ordered semigroups and monoids
- 06A15** Galois correspondences, closure operators (in relation to ordered sets)

**Keywords:**

[clones and clones](#); [Galois connection](#); [multiple-output operations](#); [partially ordered monoids](#)

**Full Text:** [DOI](#) [arXiv](#)

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