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The premium of dynamic trading in a discrete-time setting. (English) Zbl 1400.91571

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Summary: *C. H. Chiu* and *X. Y. Zhou* [Quant. Finance 11, No. 1, 115–123 (2011; [Zbl 1210.91119](#))] show that the inclusion of a risk-free asset strictly boosts the Sharpe ratio in a continuous-time setting, which is in sharp contrast to the static single-period case. In this paper, we extend their work to a discrete-time setting. Specifically, we prove that the multi-period mean-variance efficient frontier generated by both risky and risk-free assets is strictly separated from that generated by only risky assets. As a result, we demonstrate that the inclusion of a risk-free asset strictly enhances the best Sharpe ratio of the efficient frontier in a multi-period discrete-time setting. Furthermore, we offer an explicit expression for the enhancement of the best Sharpe ratio, which was referred to as the *premium of dynamic trading* by [loc. cit.], although they do not present a computational formula for it. Our results further show that, in the case with a risk-free asset, if an investor can extract some money from his initial wealth at time 0, the efficient frontier with a risk-free asset can be tangent to that without a risk-free asset. Finally, based on real data from the American market, a numerical example is provided to illustrate the results obtained in this paper; a numerical comparison between the discrete-time case and the continuous-time case is also provided. Our numerical results reveal that the continuous-time model can be considered to be a limit of the discrete-time model.

MSC:

[91G10](#) Portfolio theory

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Keywords:

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