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**Sharp one-parameter mean bounds for Yang mean.** (English) Zbl 1400.26071

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**Summary:** We prove that the double inequality  $J_\alpha(a, b) < U(a, b) < J_\beta(a, b)$  holds for all  $a, b > 0$  with  $a \neq b$  if and only if  $\alpha \leq \sqrt{2}/(\pi - \sqrt{2}) = 0.8187\dots$  and  $\beta \geq 3/2$ , where  $U(a, b) = (a - b)/[\sqrt{2} \arctan((a - b)/\sqrt{2ab})]$ , and  $J_p(a, b) = p(a^{p+1} - b^{p+1})/[(p+1)(a^p - b^p)]$  ( $p \neq 0, -1$ ),  $J_0(a, b) = (a - b)/(\log a - \log b)$ , and  $J_{-1}(a, b) = ab(\log a - \log b)/(a - b)$  are the Yang and  $p$ th one-parameter means of  $a$  and  $b$ , respectively.

**MSC:**

[26E60](#) Means

[26D07](#) Inequalities involving other types of functions

Cited in 1 Document

**Full Text:** [DOI](#)

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