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**Direct product of  $\ell$ -algebras and unification: an application to residuated lattices.** (English)

Zbl 1398.08007

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Summary: We describe classes of  $\ell$ -algebras (which are based on lattices) such that their finitely presented projective algebras are closed under finite direct product, that is, for which unification is filtering. This implies that unification in such classes is either unitary or nullary. Following ideas of *S. Ghilardi* [J. Log. Comput. 7, No. 6, 733–752 (1997; Zbl 0894.08004); J. Symb. Log. 69, No. 3, 879–906 (2004; Zbl 1069.03011)] we attempt to describe filtering unification in a variety by means of properties of factor-congruences of algebras of the variety. The results subsume some previous results, but not those of [loc. cit.], and open new areas for applications like residuated lattices. In particular we show that filtering unification depends on the monoid operation, that is, unification is filtering in varieties generated by residuated lattices without zero divisors. This implies that unification in strict fuzzy logics such as SMTL, MTL and many others is unitary or nullary.

#### MSC:

08B05 Equational logic, Mal'tsev conditions  
08A05 Structure theory of algebraic structures  
06B20 Varieties of lattices  
03G25 Other algebras related to logic

Cited in 1 Review

#### Keywords:

unification;  $\ell$ -algebra; finitely presented algebra; projective algebra; 1-regular variety; congruence kernel; central element; factor congruence; residuated lattice; zero divisors

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