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Fuzzy Galois connections on fuzzy sets. (English) Zbl 1397.03075

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Summary: In fairly elementary terms, this paper presents how the theory of preordered fuzzy sets, more precisely quantale-valued preorders on quantale-valued fuzzy sets, is established under the guidance of enriched category theory. Motivated by several key results from the theory of quantaloid-enriched categories, this paper develops all needed ingredients purely in order-theoretic languages for the readership of fuzzy set theorists, with particular attention paid to fuzzy Galois connections between preordered fuzzy sets.

MSC:

03E72 Theory of fuzzy sets, etc.

06A75 Generalizations of ordered sets

06A15 Galois correspondences, closure operators (in relation to ordered sets)

Cited in **3** Documents

Keywords:

preordered fuzzy set; fuzzy Galois connection; fuzzy relation; quantale; quantaloid

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