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A representation of antimatroids by Horn rules and its application to educational systems.

(English) [Zbl 1396.91660](#)

J. Math. Psychol. 77, 82-93 (2017).

Summary: We study a representation of an antimatroid by Horn rules, motivated by its recent application to computer-aided educational systems. We associate any set \mathcal{R} of Horn rules with the unique maximal antimatroid $\mathcal{A}(\mathcal{R})$ that is contained in the union-closed family $\mathcal{K}(\mathcal{R})$ naturally determined by \mathcal{R} . We address algorithmic and Boolean function theoretic aspects on the association $\mathcal{R} \mapsto \mathcal{A}(\mathcal{R})$, where \mathcal{R} is viewed as the input. We present linear time algorithms to solve the membership problem and the inference problem for $\mathcal{A}(\mathcal{R})$. We also provide efficient algorithms for generating all members and all implicates of $\mathcal{A}(\mathcal{R})$. We show that this representation is essentially equivalent to the Korte-Lovász representation of antimatroids by rooted sets. Based on the equivalence, we provide a quadratic time algorithm to construct the uniquely-determined minimal representation. These results have potential applications to computer-aided educational systems, where an antimatroid is used as a model of the space of possible knowledge states of learners, and is constructed by giving Horn queries to a human expert.

MSC:

[91E40](#) Memory and learning in psychology

[06A15](#) Galois correspondences, closure operators (in relation to ordered sets)

[05B35](#) Combinatorial aspects of matroids and geometric lattices

[68T30](#) Knowledge representation

Cited in **3** Documents

Keywords:

antimatroids; Horn rules; implicational systems; learning spaces; knowledge spaces; educational systems

Software:

JBool

Full Text: [DOI](#) [arXiv](#)

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