García-Lirola, Luis; Raja, Matías
Maps with the Radon-Nikodým property. (English) Zbl 1396.46013
Set-Valued Var. Anal. 26, No. 1, 77-93 (2018).

Let X be a real Banach space with dual X^*, C a closed convex subset of X and (M,d) a metric space. A function $f : C \to M$ is called dentable if, for every nonempty bounded subset A of C and every $\varepsilon > 0$, there exists an open half-space H of X such that $A \cap H \neq \emptyset$ and $\operatorname{diam}(f(A \cap H)) < \varepsilon$. Denote by $\mathcal{D}(C,M)$ the set of all dentable mappings from C to M and by $\mathcal{D}_U(C,M)$ its subset formed by all dentable mappings uniformly continuous on bounded subsets of C.

The notion is related to the Radon-Nikodým (RN) property: the set C has the RN property iff the identity mapping $I : C \to (C, \|\cdot\|)$ is dentable. Also, a continuous linear operator from X to another Banach space Y is dentable iff it is an RN operator in the sense of *O. I. Reinov* [Sov. Math., Dokl. 16, 119–123 (1975; Zbl 0317.47022); translation from Dokl. Akad. Nauk SSSR 220, 528–531 (1975)], and so the study of dentable mappings is, in some sense, a nonlinear extension of the RN property. The authors show that the set C has the RN property iff every Lipschitz mapping $f : C \to M$ is dentable. If M is a Banach space (Banach algebra, Banach lattice), then $\mathcal{D}_U(C,M)$ is also a Banach space (Banach algebra, Banach lattice, respectively) with respect to the norm of uniform convergence on C.

It is known that the strongly exposing functionals on a closed convex set with the RN property form a dense G_δ subset of X^*. The authors extend this result to this frame by replacing strongly exposing functionals by a class of functionals called strongly slicing. The possibility of uniform approximation of a uniformly continuous function f by DC (difference of convex) functions is also studied. It turns out that this happens iff the function f is finitely dentable in the sense defined by *M. Raja* [J. Convex Anal. 15, No. 2, 219–233 (2008; Zbl 1183.46018)]. Other results as, for instance, Stegall's variational principle, are no longer true beyond the usual hypotheses, sending back to the classical case.

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MSC:

46B22 Radon-Nikodým, Kreĭn-Milman and related properties
46E40 Spaces of vector- and operator-valued functions
41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)
46T20 Continuous and differentiable maps in nonlinear functional analysis

Keywords:

Radon-Nikodým property; dentability; delta-convex mappings

Full Text: DOI

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