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Gerber-Shiu analysis of a risk model with capital injections. (English) Zbl 1394.91209

Eur. Actuar. J. 6, No. 2, 409-440 (2016).

Summary: We consider the risk model with capital injections studied in our papers with *C. Nie* and *S. Li* ["Minimizing the ruin probability through capital injections", Ann. Actuar. Sci. 5, No. 2, 195–209 (2011; doi:10.1017/S1748499511000054); "The finite time ruin probability in a risk model with capital injections", Scand. Actuar. J. 2015, No. 4, 301–318 (2015; doi:10.1080/03461238.2013.823460)]. We construct a Gerber-Shiu function and show that whilst this tool is not efficient for finding the ultimate ruin probability, it provides an effective way of studying ruin related quantities in finite time. In particular, we find a general expression for the joint distribution of the time of ruin and the number of claims until ruin, and find an extension of *N. U. Prabhu*'s [Ann. Math. Stat. 32, 757–764 (1961; Zbl 0103.13302)] formula for the finite time survival probability in the classical risk model. We illustrate our results in the case of exponentially distributed claims and obtain some interesting identities. In particular, we generalise results from the classical risk model and prove the identity of two known formulae for that model.

MSC:

91B30 Risk theory, insurance (MSC2010)
60K10 Applications of renewal theory (reliability, demand theory, etc.)
62P05 Applications of statistics to actuarial sciences and financial mathematics

Keywords:

capital injections; Gerber-Shiu function; ruin probability; finite time ruin; number of claims until ruin; exponential claims

Full Text: DOI

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