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Mixed Lagrangian formulation for size-dependent couple stress elastodynamic response.

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Summary: In this paper, a mixed Lagrangian multiplier formulation is developed for consistent size-dependent couple stress elastodynamic response. This extends previous work on quasistatic couple stress response that also requires the rotation to be one half of the curl of displacement. A Lagrange multiplier, which turns out to be related to the skew-symmetric part of the force stress, is used to constrain the relation between rotation and displacement, so that rotation becomes an independent variable and C^1 continuity is avoided in the weak form. The finite element method is applied first to obtain the matrix form of the Lagrangian formulation in space, and then, discrete variational time integration is introduced to produce the discrete action. Afterward, the variation of discrete action is applied to derive the governing equations in algebraic form, which are used, along with the initial condition relations, to obtain the solutions at finite time steps. Finally, the problems of uniaxial deformation of a square plate, transverse deformation of a cantilever, and uniform traction on a square plate with a hole are investigated under this formulation, and the results are compared to existing methods where possible.

MSC:

74A10 Stress

Cited in 2 Documents

Software:

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