Conradie, Jurie; **Künzi, Hans-Peter A.**

Asymmetric norms given by symmetrisation and specialisation order. (English)

Zbl 1394.54016
Topology Appl. 242, 1-19 (2018).

A function $d : X \times X \to [0, \infty)$ of a set X is called a T_0-quasi-metric if the following conditions hold for all $x, y, z \in X$:

$$d(x, x) = 0,$$

$$d(x, z) \leq d(x, y) + d(y, z),$$

$$d(x, y) = 0 = d(y, x) \text{ implies that } x = y.$$

The authors in this paper continue the investigation between T_0-quasi-metric spaces and partially ordered metric spaces. In Section 2, they show that it is possible to set up a Galois connection between these two classes of spaces (see Proposition 2.2(c)). Furthermore, in Section 3, the authors establish a relationship between nonexpansive maps between T_0-quasi-metric spaces and nonexpansive and increasing maps between partially ordered metric spaces (see Propositions 3.1 and 3.2).

Finally, in the last section (Section 4), they derive a representation theorem for injective asymmetrically normed spaces. This answers a question that was left open in [*J. Conradie* et al., Topology Appl. 231, 92–112 (2017; Zbl 1387.46050)].

Reviewer: Collins Agyingi (Mmabatho)

MSC:

54E35 Metric spaces, metrizability
46M10 Projective and injective objects in functional analysis
46B40 Ordered normed spaces
54F05 Linearly ordered topological spaces, generalized ordered spaces, and partially ordered spaces
06A15 Galois correspondences, closure operators (in relation to ordered sets)

Keywords:

T_0-quasi-metric; asymmetric norm; injective; specialisation order; Galois connection; maximal quasi-metric; producing quasi-metric

Full Text: DOI

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