Miller, Jason; **Sheffield, Scott**

Imaginary geometry. III: Reversibility of SLE_κ for $\kappa \in (4,8)$. (English) Zbl 1393.60092

Ann. Math. (2) 184, No. 2, 455-486 (2016).

From the text: Fix $k \in (2,4)$, and write $k' = 16/k \in (4,8)$. Our main result is the following:

Theorem 1.1. Suppose that D is a Jordan domain, and let $x, y \in \partial D$ be distinct. Let η' be a chordal $\mathrm{SLE}_{k'}$ process in D from x to y. Then the law of η' has time-reversal symmetry. That is, if $\psi : D \to D$ is an anti-conformal map that swaps x and y, then the time-reversal of $\psi \circ \eta'$ is equal in law to η', up to reparametrization.

Theorem 1.1 is a special case of a more general theorem that gives the time-reversal symmetry of $\mathrm{SLE}_{k'}(\rho_1, \rho_2)$ processes provided $\rho_1, \rho_2 \geqslant k'/2 - 4$.

Theorem 1.2. Suppose that D is a Jordan domain, and let $x, y \in \partial D$ be distinct. Suppose that η' is a chordal $\mathrm{SLE}_{k'}(\rho_1, \rho_2)$ process in D from x to y where the force points are located at x^- and x^+. If $\psi : D \to D$ is an anti-conformal map that swaps x and y, then the time-reversal of $\psi \circ \eta'$ is an $\mathrm{SLE}_{k'}(\rho_1, \rho_2)$ process from x to y, up to reparametrization.

Our final result is the nonreversibility of $\mathrm{SLE}_{k'}(\rho_1, \rho_2)$ processes when either $\rho_1 < k'/2 - 4$ or $\rho_2 < k'/2 - 4$.

Theorem 3.1. Suppose that D is a Jordan domain, and let $x, y \in \partial D$ be distinct. Suppose that η' is a chordal $\mathrm{SLE}_{k'}(\rho_1, \rho_2)$ process in D from x to y. Let $\psi : D \to D$ be an anti-conformal map that swaps x and y. If either $\rho_1 < k'/2 - 4$ or $\rho_2 < k'/2 - 4$, then the law of the time-reversal of $\psi(\eta')$ is not an $\mathrm{SLE}_{k'}(\rho)$ process for any collection of weights ρ.

For Part I and Part II see [the authors, Probab. Theory Relat. Fields 164, No. 3–4, 553–705 (2016; Zbl 1336.60162); Ann. Probab. 44, No. 3, 1647–1722 (2016; Zbl 1344.60078)].

Reviewer: Oleg K. Zakusilo (Kyïv)

MSC:

60J67 Stochastic (Schramm-)Loewner evolution (SLE)
60G60 Random fields
60K35 Interacting random processes; statistical mechanics type models; percolation theory
60G15 Gaussian processes
60D05 Geometric probability and stochastic geometry

Keywords:

conformal loop ensembles; Gaussian free field; reversibility; Schramm-Loewner evolution

Full Text: DOI

References:

[1] G. Ben Arous and J. -D. Deuschel, "The construction of the \((d+1)\)-dimensional Gaussian droplet," Comm. Math. Phys., vol. 179, iss. 2, pp. 467-488, 1996. · Zbl 0858.60096 · doi:10.1007/BF02102597

[2] F. Camia and C. M. Newman, "Two-dimensional critical percolation: the full scaling limit," Comm. Math. Phys., vol. 268, iss. 1, pp. 1-38, 2006. · Zbl 1117.60086 · doi:10.1007/s00220-006-0086-1

[3] D. Chelkak and S. Smirnov, "Universality in the 2D Ising model and conformal invariance of fermionic observables," Invent. Math., vol. 189, iss. 3, pp. 515-580, 2012. · Zbl 1257.82020 · doi:10.1007/s00222-011-0371-2

[4] J. Dubédat, "Commutation relations for Schramm-Loewner evolutions," Comm. Pure Appl. Math., vol. 60, iss. 12, pp. 1792-1847, 2007. · Zbl 1137.82009 · doi:10.1002/cpa.20191

[5] J. Dubédat, "Duality of Schramm-Loewner evolutions," Ann. Sci. Éc. Norm. Supér., vol. 42, iss. 5, pp. 697-724, 2009. · Zbl 1205.60147

[6] J. Dubédat, "SLE and the free field: partition functions and couplings," J. Amer. Math. Soc., vol. 22, iss. 4, pp. 995-1054, 2009. · Zbl 1204.60079 · doi:10.1090/S0894-0347-09-00636-5

[7] C. Hagendorf, D. Bernard, and M. Bauer, "The Gaussian free field and \({ SLE}_4\) on doubly connected domains," J. Stat. Phys., vol. 140, iss. 1, pp. 1-26, 2010. · Zbl 1193.82027 · doi:10.1007/s10955-010-9980-1

[8] K. Izyurov and K. Kytölä, "Hadamard's formula and couplings of SLEs with free field," Probab. Theory Related Fields, vol. 155, iss. 1-2, pp. 35-69, 2013. · Zbl 1269.60067 · doi:10.1007/s00440-011-0391-2

[9] R. Kenyon, "Dominos and the Gaussian free field," Ann. Probab., vol. 29, iss. 3, pp. 1128-1137, 2001. · Zbl 1034.82021 · doi:10.1214/aop/1015345599

[10] G. F. Lawler, Conformally Invariant Processes in the Plane, Providence, RI: Amer. Math. Soc., 2005, vol. 114. · Zbl 1074.60002

[11] G. F. Lawler, O. Schramm, and W. Werner, "Conformal invariance of planar loop-erased random walks and uniform spanning trees," Ann. Probab., vol. 32, iss. 1B, pp. 939-995, 2004. · Zbl 1126.82011 · doi:10.1214/aop/1079021469

[12] J. Miller, Universality for SLE(4), 2010.

[13] J. Miller, "Fluctuations for the Ginzburg-Landau \(\nabla\phi\) interface model on a bounded domain," Comm. Math. Phys., vol. 308, iss. 3, pp. 591-639, 2011. · Zbl 1237.82030 · doi:10.1007/s00220-011-1315-9

[14] N. Makarov and S. Smirnov, "Off-critical lattice models and massive SLEs," in XVIth International Congress on Mathematical Physics, Hackensack, NJ: World Sci. Publ., 2010, pp. 362-371. · Zbl 1205.82055 · doi:10.1142/9789814304634$_0$024

[15] J. Miller and S. Sheffield, Imaginary geometry I: Interacting SLEs, 2012. · Zbl 1336.60162 · doi:10.1007/s00440-016-0698-0

[16] J. Miller and S. Sheffield, Imaginary geometry II: Reversibility of SLE\(_\kappa(\rho_1;\rho_2)\) for \(\kappa \in (0,4)\), 2012. · Zbl 1344.60078

[17] A. Naddaf and T. Spencer, "On homogenization and scaling limit of some gradient perturbations of a massless free field," Comm. Math. Phys., vol. 183, iss. 1, pp. 55-84, 1997. · Zbl 0871.35010 · doi:10.1007/BF02509796

[18] S. Rohde and O. Schramm, "Basic properties of SLE," Ann. of Math., vol. 161, iss. 2, pp. 883-924, 2005. · Zbl 1081.60069 · doi:10.4007/annals.2005.161.883

[19] B. Rider and B. Virág, "The noise in the circular law and the Gaussian free field," Int. Math. Res. Not., vol. 2007, iss. 2, p. I, 2007. · Zbl 1130.60030 · doi:10.1093/imrn/rnm006

[20] O. Schramm, "Scaling limits of loop-erased random walks and uniform spanning trees," Israel J. Math., vol. 118, pp. 221-288, 2000. · Zbl 0968.60093 · doi:10.1007/BF02803524

[21] O. Schramm, "A percolation formula," Electron. Comm. Probab., vol. 6, pp. 115-120, 2001. · Zbl 1008.60100 · doi:10.1214/ECP.v6-1041

[22] S. Sheffield, Local sets of the Gaussian free field: slides and audio. · Zbl 1132.60072

[23] S. Sheffield, "Exploration trees and conformal loop ensembles," Duke Math. J., vol. 147, iss. 1, pp. 79-129, 2009. · Zbl 1170.60008 · doi:10.1215/00127094-2009-007

[24] S. Sheffield, Conformal weldings of random surfaces: SLE and the quantum gravity zipper, 2010. · Zbl 1388.60144

[25] S. Smirnov, "Critical percolation in the plane: conformal invariance, Cardy's formula, scaling limits," C. R. Acad. Sci. Paris Sér. I Math., vol. 333, iss. 3, pp. 239-244, 2001. · Zbl 0985.60090 · doi:10.1016/S0764-4442(01)01991-7

[26] S. Smirnov, "Conformal invariance in random cluster models. I. Holomorphic fermions in the Ising model," Ann. of Math., vol. 172, iss. 2, pp. 1435-1467, 2010. · Zbl 1200.82011 · doi:10.4007/annals.2010.172.1441

[27] O. Schramm and S. Sheffield, "Harmonic explorer and its convergence to \({ SLE}_4\)," Ann. Probab., vol. 33, iss. 6, pp. 2127-2148, 2005. · Zbl 1095.60007 · doi:10.1214/009117905000000477

[28] O. Schramm and S. Sheffield, "Contour lines of the two-dimensional discrete Gaussian free field," Acta Math., vol. 202, iss. 1, pp. 21-137, 2009. · Zbl 1210.60051 · doi:10.1007/s11511-009-0034-y

[29] O. Schramm and S. Sheffield, "A contour line of the continuum Gaussian free field," Probab. Theory Related Fields, vol. 157, iss. 1-2, pp. 47-80, 2013. · Zbl 1331.60090 · doi:10.1007/s00440-012-0449-9

[30] S. Sheffield and W. Werner, "Conformal loop ensembles: the Markovian characterization and the loop-soup construction," Ann. of Math., vol. 176, iss. 3, pp. 1827-1917, 2012. · Zbl 1271.60090 · doi:10.4007/annals.2012.176.3.8

[31] W. Werner, "Random planar curves and Schramm-Loewner evolutions," in Lectures on Probability Theory and Statistics, New York: Springer-Verlag, 2004, vol. 1840, pp. 107-195. · Zbl 1057.60078 · doi:10.1007/978-3-540-39982-7$_2$

[32] D. Zhan, "Reversibility of chordal SLE," Ann. Probab., vol. 36, iss. 4, pp. 1472-1494, 2008. · Zbl 1157.60051 · doi:10.1214/07-AOP366

[33] D. Zhan, "Reversibility of some chordal \({ SLE}(\kappa;\rho)\) traces," J. Stat. Phys., vol. 139, iss. 6, pp. 1013-1032, 2010. · Zbl 1205.82063 · doi:10.1007/s10955-010-9978-8