Maddix, Danielle C.; **Sampaio, Luiz**; **Gerritsen, Margot**

Numerical artifacts in the generalized porous medium equation: why harmonic averaging itself is not to blame. (English) ⊡ Zbl 1391.76734

Summary: The degenerate parabolic generalized porous medium equation (GPME) poses numerical challenges due to self-sharpening and its sharp corner solutions. For these problems, we show results for two subclasses of the GPME with differentiable $k(p)$ with respect to p, namely the porous medium equation (PME) and the superslow diffusion equation. Spurious temporal oscillations, and nonphysical locking and lagging have been reported in the literature. These issues have been attributed to harmonic averaging of the coefficient $k(p)$ for small p, and arithmetic averaging has been suggested as an alternative. We show that harmonic averaging is not solely responsible and that an improved discretization can mitigate these issues. Here, we investigate the causes of these numerical artifacts using modified equation analysis. The modified equation framework can be used for any type of discretization. We show results for the second order finite volume method. The observed problems with harmonic averaging can be traced to two leading error terms in its modified equation. This is also illustrated numerically through a modified harmonic method (MHM) that can locally modify the critical terms to remove the aforementioned numerical artifacts.

MSC:

76S05	Flows in porous media; filtration; seepage
76M25	Other numerical methods (fluid mechanics) (MSC2010)
35K65	Degenerate parabolic equations

Cited in **1** Review
Cited in **2** Documents

Keywords:

generalized porous medium equation; degenerate nonlinear parabolic equations; harmonic and arithmetic averaging; modified equation analysis; temporal oscillations; self-sharpening

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