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**Categorical aspects of inducing closure operators on graphs by sets of walks.** (English)

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Summary: We study closure operators on graphs which are induced by sets of walks of identical lengths in these graphs. It is shown that the induction gives rise to a Galois correspondence between the category of closure spaces and that of graphs with walk sets. We study the two isomorphic subcategories resulting from the correspondence, in particular, the one that is a full subcategory of the category of graphs with walk sets. As examples, we discuss closure operators that are induced by path sets on some natural graphs on the digital plane  $\mathbb{Z}^2$ . These closure operators are shown to include the well known Marcus-Wyse and Khalimsky topologies, thus indicating the possibility of using them as convenient background structures on the digital plane for the study of geometric and topological properties of digital images.

**MSC:**

- [68U05](#) Computer graphics; computational geometry (digital and algorithmic aspects)
- [06A15](#) Galois correspondences, closure operators (in relation to ordered sets)
- [18B99](#) Special categories
- [54A05](#) Topological spaces and generalizations (closure spaces, etc.)
- [68R10](#) Graph theory (including graph drawing) in computer science

**Keywords:**

[simple graph](#); [walk](#); [closure operator](#); [Galois correspondence between concrete categories](#); [diagonal walk set](#); [digital topology](#)

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