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Direct arbitrary-Lagrangian-Eulerian finite volume schemes on moving nonconforming unstructured meshes. (English) Zbl 1390.76433

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Summary: In this paper, we present a novel second-order accurate arbitrary-Lagrangian-Eulerian (ALE) finite volume scheme on moving nonconforming polygonal grids, in order to avoid the typical mesh distortion caused by shear flows in Lagrangian-type methods. In our new approach the nonconforming element interfaces are not defined by the user, but they are automatically detected by the algorithm if the tangential velocity difference across an element interface is sufficiently large. The grid nodes that are sufficiently far away from a shear wave are moved with a standard node solver, while at the interface we insert a new set of nodes that can slide along the interface in a nonconforming manner. In this way, the elements on both sides of the shear wave can move with a different velocity, without producing highly distorted elements. The core of the proposed method is the use of a space-time conservation formulation in the construction of the final finite volume scheme, which completely avoids the need of an additional remapping stage, hence the new method is a so-called direct ALE scheme. For this purpose, the governing PDE system is rewritten at the aid of the space-time divergence operator and then a fully discrete one-step discretization is obtained by integrating over a set of closed space-time control volumes. The nonconforming sliding of nodes along an edge requires the insertion or the deletion of nodes and edges, and in particular the space-time faces of an element can be shared between more than two cells. Due to the space-time conservation formulation, the geometric conservation law (GCL) is automatically satisfied by construction, even on moving nonconforming meshes. Moreover, the mesh quality remains high and, as a direct consequence, also the time step remains almost constant in time, even for highly sheared vortex flows. In this paper we focus mainly on logically straight slip-line interfaces, but we show also first results for general slide lines that are not logically straight. Second order of accuracy in space and time is obtained by using a MUSCL-Hancock strategy, together with a Barth and Jespersen slope limiter. The accuracy of the new scheme has been further improved by incorporating a special well balancing technique that is able to maintain particular stationary solutions of the governing PDE system up to machine precision. In particular, we consider steady vortex solutions of the shallow water equations, where the pressure gradient is in equilibrium with the centrifugal forces. A large set of different numerical tests has been carried out in order to check the accuracy and the robustness of the new method for both smooth and discontinuous problems. In particular, we have compared the results for a steady vortex in equilibrium solved with a standard conforming ALE method (without any rezoning technique) and with our new nonconforming ALE scheme, to show that the new nonconforming scheme is able to avoid mesh distortion in vortex flows even after very long simulation times.

MSC:

76M12	Finite volume methods applied to problems in fluid mechanics
65M08	Finite volume methods for initial value and initial-boundary value problems involving PDEs

Keywords:

moving nonconforming unstructured meshes; slide lines in direct arbitrary-Lagrangian-Eulerian (ALE) methods for shear flows; cell-centered Godunov-type finite volume methods; shallow water equations in Cartesian and cylindrical coordinates; hyperbolic conservation laws; well balanced methods

Software:

ReALE

Full Text: DOI

References:

[1] von Neumann, J.; Richtmyer, R., A method for the calculation of hydrodynamics shocks, J Appl Phys, 21, 232-237, (1950) · Zbl 0037.12002

[2] Wilkins, M. L., Calculation of elastic-plastic flow, Meth Comput Phys, 3, (1964)

[3] Caramana, E.; Burton, D.; Shashkov, M.; Whalen, P., The construction of compatible hydrodynamics algorithms utilizing conservation of total energy, J Comput Phys, 146, 227-262, (1998) · Zbl 0931.76080

[4] Caramana, E.; Shashkov, M., Elimination of artificial grid distorsion and hourglass type motions by means of Lagrangian subzonal masses and pressures, J Comput Phys, 142, 521-561, (1998) · Zbl 0932.76068

[5] Loubère, R.; Maire, P.; Váchal, P., A second-order compatible staggered Lagrangian hydrodynamics scheme using a cell-centered multidimensional approximate Riemann solver, Procedia Comput Sci, 1, 1931-1939, (2010) · Zbl 1432.76206

[6] Loubère, R.; Maire, P.; Váchal, P., 3D staggered Lagrangian hydrodynamics scheme with cell-centered Riemann solver-based artificial viscosity, Int J Numer Methods Fluids, 72, 22-42, (2013)

[7] Loubère, R.; Shashkov, M., A subcell remapping method on staggered polygonal grids for arbitrary-Lagrangian-Eulerian methods, J Comput Phys, 23, 155-160, (2004)

[8] Godunov, S., Finite difference methods for the computation of discontinuous solutions of the equations of fluid dynamics, Math USSR: Sbornik, 47, 271-306, (1959) · Zbl 0171.46204

[9] Munz, C., On Godunov-type schemes for Lagrangian gas dynamics, SIAM J Numer Anal, 31, 17-42, (1994) · Zbl 0796.76057

[10] Després, B.; Mazeran, C., Symmetrization of Lagrangian gas dynamic in dimension two and multidimensional solvers, CR Mecanique, 331, 475-480, (2003) · Zbl 1293.76089

[11] Després, B.; Mazeran, C., Lagrangian gas dynamics in two-dimensions and Lagrangian systems, Arch Ration Mech Anal, 178, 327-372, (2005) · Zbl 1096.76046

[12] Carré, G.; Pino, S. D.; Després, B.; Labourasse, E., A cell-centered Lagrangian hydrodynamics scheme on general unstructured meshes in arbitrary dimension., J Comput Phys, 228, 5160-5183, (2009) · Zbl 1168.76029

[13] Maire, P.; Abgrall, R.; Breil, J.; Ovadia, J., A cell-centered Lagrangian scheme for two-dimensional compressible flow problems, SIAM J Sci Comput, 29, 1781-1824, (2007) · Zbl 1251.76028

[14] Maire, P., A high-order cell-centered Lagrangian scheme for two-dimensional compressible fluid flows on unstructured meshes., J Comput Phys, 228, 2391-2425, (2009) · Zbl 1156.76434

[15] Maire, P., A high-order one-step sub-cell force-based discretization for cell-centered Lagrangian hydrodynamics on polygonal grids, Comput Fluids, 46(1), 341-347, (2011) · Zbl 1433.76137

[16] Maire, P., A unified sub-cell force-based discretization for cell-centered Lagrangian hydrodynamics on polygonal grids, Int J Numer Methods Fluids, 65, 1281-1294, (2011) · Zbl 1429.76089

[17] Cheng, J.; Shu, C., A high order ENO conservative Lagrangian type scheme for the compressible Euler equations, J Comput Phys, 227, 1567-1596, (2007) · Zbl 1126.76035

[18] Liu, W.; Cheng, J.; Shu, C., High order conservative Lagrangian schemes with Lax-Wendroff type time discretization for the compressible Euler equations, J Comput Phys, 228, 8872-8891, (2009) · Zbl 1287.76181

[19] Dumbser, M.; Uuriintsetseg, A.; Zanotti, O., On arbitrary-Lagrangian-Eulerian one-step WENO schemes for stiff hyperbolic balance laws, Commun Comput Phys, 14, 301-327, (2013) · Zbl 1373.76126

[20] Cheng, J.; Toro, E., A 1D conservative Lagrangian ADER scheme, Chin J Comput Phys, 30, 501-508, (2013)

[21] Vilar, F.; Maire, P.; Abgrall, R., Cell-centered discontinuous Galerkin discretizations for two-dimensional scalar conservation laws on unstructured grids and for one-dimensional Lagrangian hydrodynamics, Comput Fluids, 46(1), 498-604, (2010) · Zbl 1433.76093

[22] Vilar, F., Cell-centered discontinuous Galerkin discretization for two-dimensional Lagrangian hydrodynamics, Comput Fluids, 64, 64-73, (2012) · Zbl 1365.76129

[23] Vilar, F.; Maire, P.; Abgrall, R., A discontinuous Galerkin discretization for solving the two-dimensional gas dynamics equations written under total Lagrangian formulation on general unstructured grids, J Comput Phys, 276, 188-234, (2014) · Zbl 1349.76278

[24] Li, Z.; Yu, X.; Jia, Z., The cell-centered discontinuous Galerkin method for Lagrangian compressible Euler equations in two dimensions, Comput Fluids, 96, 152-164, (2014) · Zbl 1391.76347

[25] Boscheri, W.; Dumbser, M., Arbitrary-Lagrangian-Eulerian discontinuous Galerkin schemes with a posteriori subcell finite volume limiting on moving unstructured meshes, J Comput Phys, 346, 449-479, (2017) · Zbl 1378.76044

[26] Ortega, A. L.; Scovazzi, G., A geometrically-conservative, synchronized, flux-corrected remap for arbitrary Lagrangian-Eulerian computations with nodal finite elements, J Comput Phys, 230, 6709-6741, (2011) · Zbl 1284.76255

[27] Scovazzi, G., Lagrangian shock hydrodynamics on tetrahedral meshes: a stable and accurate variational multiscale approach, J Comput Phys, 231, 8029-8069, (2012)

[28] Dobrev, V.; Ellis, T.; Kolev, T.; Rieben, R., Curvilinear finite elements for Lagrangian hydrodynamics, Int J Numer Methods Fluids, 65, 1295-1310, (2011) · Zbl 1255.76075

[29] Dobrev, V.; Ellis, T.; Kolev, T.; Rieben, R., High order curvilinear finite elements for Lagrangian hydrodynamics, SIAM J Sci Comput, 34, 606-641, (2012)

[30] Dobrev, V.; Ellis, T.; Kolev, T.; Rieben, R., High order curvilinear finite elements for axisymmetric Lagrangian hydrodynamics, Computers and Fluids, 83, 58-69, (2013) · Zbl 1290.76061

[31] Balsara, D., A two-dimensional HLLC Riemann solver for conservation laws: application to Euler and magnetohydrodynamic

flows, J Comput Phys, 231, 7476-7503, (2012) · Zbl 1284.76261

[32] Balsara, D.; Dumbser, M.; Abgrall., R., Multidimensional HLLC Riemann solver for unstructured meshes - with application to Euler and MHD flows, J Comput Phys, 261, 172-208, (2014) · Zbl 1349.76426

[33] Balsara, D., Multidimensional Riemann problem with self-similar internal structure part I application to hyperbolic conservation laws on structured meshes, J Comput Phys, 277, 163-200, (2014) · Zbl 1349.76303

[34] Balsara, D.; Dumbser, M., Multidimensional Riemann problem with self-similar internal structure part II application to hyperbolic conservation laws on unstructured meshes, J Comput Phys, 287, 269-292, (2015) · Zbl 1351.76091

[35] Boscheri, W.; Dumbser, M.; Balsara, D., High order Lagrangian ADER-WENO schemes on unstructured meshes-application of several node solvers to hydrodynamics and magnetohydrodynamics, Int J Numer Methods Fluids, 76, 737-778, (2014)

[36] Berndt, M.; Breil, J.; Galera, S.; Kucharik, M.; Maire, P.; Shashkov, M., Two-step hybrid conservative remapping for multimaterial arbitrary Lagrangian-Eulerian methods, J Comput Phys, 230, 6664-6687, (2011) · Zbl 1408.65077

[37] Bochev, P.; Ridzal, D.; Shashkov, M., Fast optimization-based conservative remap of scalar fields through aggregate mass transfer, J Comput Phys, 246, 37-57, (2013) · Zbl 1349.65054

[38] Breil, J.; Harribey, T.; Maire, P.; Shashkov, M., A multi-material reale method with MOF interface reconstruction, Comput Fluids, 83, 115-125, (2013) · Zbl 1290.76094

[39] Kucharik, M.; Shashkov, M., One-step hybrid remapping algorithm for multi-material arbitrary Lagrangian-Eulerian methods, J Comput Phys, 231, 2851-2864, (2012) · Zbl 1323.74108

[40] Yanilkin, Y.; Goncharov, E.; Kolobyanin, V.; Sadchikov, V.; Kamm, J.; Shashkov, M., Multi-material pressure relaxation methods for Lagrangian hydrodynamics., Comput Fluids, 83, 137-143, (2013) · Zbl 1290.76138

[41] Loubère, R.; Dumbser, M.; Diot, S., A new family of high order unstructured MOOD and ADER finite volume schemes for multidimensional systems of hyperbolic conservation law, Commun Comput Phys, 16, 718-763, (2014) · Zbl 1373.76137

[42] Blanchard, G.; Loubère, R., High order accurate conservative remapping scheme on polygonal meshes using a posteriori MOOD limiting, Comput Fluids, 136, 83-103, (2016) · Zbl 1390.65101

[43] Boscheri, W.; Balsara, D.; Dumbser, M., Lagrangian ADER-WENO finite volume schemes on unstructured triangular meshes based on genuinely multidimensional HLL Riemann solvers, J Comput Phys, 267, 112-138, (2014) · Zbl 1349.76309

[44] Boscheri, W.; Dumbser, M., Arbitrary-Lagrangian-Eulerian one-step WENO finite volume schemes on unstructured triangular meshes, Commun Comput Phys, 14, 1174-1206, (2013) · Zbl 1388.65075

[45] Boscheri, W.; Dumbser, M., A direct arbitrary-Lagrangian-Eulerian ADER-WENO finite volume scheme on unstructured tetrahedral meshes for conservative and non-conservative hyperbolic systems in 3d, J Comput Phys, 275, 484-523, (2014) · Zbl 1349.76310

[46] Kucharik, M.; Liska, R.; Limpouch, J.; Váchal, P., ALE simulations of high-velocity impact problem, J Comput Phys, 76, 737-778, (2014)

[47] Caramana, E., The implementation of slide lines as a combined force and velocity boundary condition, J Comput Phys, 228, 3911-3916, (2009) · Zbl 1273.76258

[48] Barlow, A.; Whittle, J., Mesh adaptivity and material interface algorithms in a two dimensional Lagrangian hydrocode, Chem Phys, 19, 15-26, (2000)

[49] Kucharik, M.; Loubère, R.; Bednàrik, L.; Liska, R., Enhancement of Lagrangian slide lines as a combined force and velocity boundary condition, Comput Fluids, 83, 3-14, (2013) · Zbl 1290.76137

[50] Clair, G.; Despres, B.; Labourasse, E., A new method to introduce constraints in cell-centered Lagrangian schemes, Comput Methods Appl Mech Eng, 261-262, 56-65, (2013) · Zbl 1286.76111

[51] Clair, G.; Despres, B.; Labourasse, E., A one-mesh method for the cell-centered discretization of sliding, Comput Methods Appl Mech Eng, 269, 315-333, (2014) · Zbl 1296.76131

[52] Pino, S. D., A curvilinear finite-volume method to solve compressible gas dynamics in semi-Lagrangian coordinates, Comptes Rendus de l'Académie des Sciences - Series I - Mathematics, 348, 1027-1032, (2010) · Zbl 1426.76652

[53] Bertoluzza, S.; Pino, S. D.; Labourasse, E., A conservative slide line method for cell-centered semi-Lagrangian and ALE schemes in 2D, ESAIM, 50, 187-214, (2016) · Zbl 1382.76181

[54] Springel, V., E pur si muove: Galilean-invariant cosmological hydrodynamical simulations on a moving mesh, Mon Not R Astron Soc, 401, 2, 791-851, (2010)

[55] Dumbser, M., Arbitrary-Lagrangian-Eulerian ADER-WENO finite volume schemes with time-accurate local time stepping for hyperbolic conservation laws, Comput Methods Appl Mech Eng, 280, 57-83, (2014) · Zbl 1423.76296

[56] Boscheri, W.; Dumbser, M.; Zanotti, O., High order cell-centered Lagrangian-type finite volume schemes with time-accurate local time stepping on unstructured triangular meshes, J Comput Phys, 291, 120-150, (2015) · Zbl 1349.76311

[57] Cavalcanti, J.; Dumbser, M.; da Motta-Marques, D.; Junior, C. F., A conservative finite volume scheme with time-accurate local time stepping for scalar transport on unstructured grids, Adv Water Resour, 86, 217-230, (2015)

[58] Parés, C., Numerical methods for nonconservative hyperbolic systems: a theoretical framework, SIAM J Numer Anal, 44, 300-321, (2006) · Zbl 1130.65089

[59] van Leer, B., Towards the ultimate conservative difference scheme II: monotonicity and conservation combined in a second order scheme, J Comput Phys, 14, 361-370, (1974) · Zbl 0276.65055

[60] Toro, E., Riemann solvers and numerical methods for fluid dynamics, (1999), Springer

[61] Barth, T.; Jespersen, D., The design and application of upwind schemes on unstructured meshes, AIAA Paper 89-0366, 1-12,

(1989)

[62] Dumbser, M.; Toro, E. F., On universal osher-type schemes for general nonlinear hyperbolic conservation laws, Commun Comput Phys, 10, 635-671, (2011) · Zbl 1373.76125

[63] Castro, M.; Gallardo, J.; Parés, C., High-order finite volume schemes based on reconstruction of states for solving hyperbolic systems with nonconservative products. applications to shallow-water systems, Math Comput, 75, 1103-1134, (2006) · Zbl 1096.65082

[64] Botta, N.; Klein, R.; Langenberg, S.; Lützenkirchen, S., Well-balanced finite volume methods for nearly hydrostatic flows, J Comput Phys, 196, 539-565, (2004) · Zbl 1109.86304

[65] Chandrashekar, P.; Klingenberg, C., A second order well-balanced finite volume scheme for Euler equations with gravity, SIAM J Sci Comput, 37, B382-B402, (2015) · Zbl 1320.76078

[66] Käppeli, R.; Mishra, S., Well-balanced schemes for the Euler equations with gravitation, J Comput Phys, 259, 199-219, (2014) · Zbl 1349.76345

[67] Gallardo, J.; Parés, C.; Castro, M., On a well-balanced high-order finite volume scheme for shallow water equations with topography and dry areas, J Comput Phys, 227, 574-601, (2007) · Zbl 1126.76036

[68] Dal Maso, G.; LeFloch, P.; Murat, F., Definition and weak stability of nonconservative products, J Math Pures Appl, 74, 483-548, (1995) · Zbl 0853.35068

[69] Dumbser, M.; Toro, E. F., A simple extension of the osher Riemann solver to non-conservative hyperbolic systems, J Sci Comput, 48, 70-88, (2011) · Zbl 1220.65110

[70] Castro, M.; Gallardo, J.; López-García, J.; Parés, C., Well-balanced high order extensions of godunov's method for semi-linear balance laws, SIAM J Numer Anal, 46, 2, 1012-1039, (2008) · Zbl 1159.74045