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Kink dynamics in the ϕ^4 model: asymptotic stability for odd perturbations in the energy space. (English) Zbl 1387.35419

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The authors consider the one-dimensional ϕ^4 model,

$$\partial_t^2\phi - \partial_x^2\phi = \phi - \phi^3, \qquad (t,x) \in \mathbb{R}\times\mathbb{R}$$

and the examine the stability of odd perturbations to the *kink* stationary solution

$$\phi^*(x) = \tanh\left(\frac{x}{\sqrt{2}}\right).$$

The main result shows the asymptotic stability of the kink with respect to odd perturbations in the energy space. The result joins the proof of the orbital stability of the kink with respect to small perturbations [*D. B. Henry* et al., Commun. Math. Phys. 85, 351–361 (1982; Zbl 0546.35062)] to describe the long time behavior of solutions.

Reviewer: Joseph Shomberg (Providence)

MSC:

35L71	Second-order semilinear hyperbolic equations
35Q51	Soliton equations
37K40	Soliton theory, asymptotic behavior of solutions of infinite-dimensional Hamiltonian systems
35B35	Stability in context of PDEs
35L15	Initial value problems for second-order hyperbolic equations

Keywords:

one space dimension; Virial-type estimates; internal oscillation mode

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