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Basic theorem of fuzzy concept lattices revisited. (English) Zbl 1385.06006
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Authors' abstract: There are two versions of the basic theorem of L -concept lattices for L being a complete residuated lattice, both proved by *R. Bělohlávek*: the crisp-order version [*Math. Log. Q.* 47, No. 1, 111–116 (2001; [Zbl 0976.03025](#))] and the fuzzy-order version [*Ann. Pure Appl. Logic* 128, No. 1–3, 277–298 (2004; [Zbl 1060.03040](#))]. In this paper, the author introduces a third version, equivalent to the fuzzy-order version, but simpler and related more closely to the classical basic theorem of concept lattices by *R. Wille* [in: *Ordered sets. Proceedings of the NATO Advanced Study Institute held at Banff, Canada, 1981.* Dordrecht-Boston-London: D. Reidel Publishing Company. 445–470 (1982; [Zbl 0491.06008](#))]. Then, he uses it to prove some new results on substructures of L -concept lattices and shows a simpler proof of a known result on factor structures of L -concept lattices. He shows by means of several counterexamples that the crisp order version does not describe the structure of L -concept lattices sufficiently. He argues that in order to formulate and prove theoretical results on L -concept lattices that are similar to those known from classical formal concept analysis, it is essential to use the fuzzy order version of the basic theorem. He also discusses the correspondence between Belohlavek's [2004, loc. cit.] fuzzy-order version of the basic theorem and the version introduced in this paper.

Reviewer: [Fu-Gui Shi \(Beijing\)](#)

MSC:

[06D72](#) Fuzzy lattices (soft algebras) and related topics
[06A15](#) Galois correspondences, closure operators (in relation to ordered sets)
[06B23](#) Complete lattices, completions

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[fuzzy relations](#); [fuzzy order](#); [formal concept analysis](#); [basic theorem of concept lattices](#)

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References:

- [1] Belohlavek, R., Fuzzy Galois connections, *Math. Log. Q.*, 45, 4, 497-504, (1999) · [Zbl 0938.03079](#)
- [2] Belohlavek, R., Lattices of fixed points of fuzzy Galois connections, *Math. Log. Q.*, 47, 1, 111-116, (2001) · [Zbl 0976.03025](#)
- [3] Belohlavek, R., Concept lattices and order in fuzzy logic, *Ann. Pure Appl. Log.*, 128, 1-3, 277-298, (2004) · [Zbl 1060.03040](#)
- [4] Belohlavek, R., Fuzzy relational systems: foundations and principles, (2002), Kluwer Academic Publishers Norwell, USA · [Zbl 1067.03059](#)
- [5] Belohlavek, R., What is a fuzzy concept lattice? II, (Kuznetsov, S.; Slezak, D.; Hepting, D. H.; Mirkin, B. G., *Rough Sets, Fuzzy Sets, Data Mining and Granular Computing, Lect. Notes Comput. Sci.*, vol. 6743, (2011), Springer Berlin Heidelberg), 19-26
- [6] Belohlavek, R.; Sklenar, V.; Zacpal, J., Crisply generated fuzzy concepts, (Ganter, B.; Godin, R., *Formal Concept Analysis, Lect. Notes Comput. Sci.*, vol. 3403, (2005), Springer Berlin, Heidelberg), 269-284 · [Zbl 1078.68142](#)
- [7] Belohlavek, R.; Vychodil, V., Reducing the size of fuzzy concept lattices by hedges, (*Proceedings of FUZZ-IEEE 2005: The 14th IEEE International Conference on Fuzzy Systems*, (2005)), 663-668
- [8] Belohlavek, R.; Vychodil, V., What is a fuzzy concept lattice?, (Belohlavek, R.; Snasel, V., *Proc. CLA 2005*, (2005)), 34-45
- [9] Burusco, A.; Fuentes-González, R., The study of the L -fuzzy concept lattice, *Mathw. Soft Comput.*, 3, 209-218, (1994) · [Zbl 0827.06004](#)
- [10] Czédli, G., Factor lattices by tolerances, *Acta Sci. Math.*, 44, 35-42, (1982) · [Zbl 0484.06010](#)
- [11] Fan, L., A new approach to quantitative domain theory, MFPS 2001, Seventeenth Conference on the Mathematical Foundations of Programming Semantics, *Electron. Notes Theor. Comput. Sci.*, 45, 77-87, (2001) · [Zbl 1260.68217](#)
- [12] Ganter, B.; Wille, R., *Formal concept analysis - mathematical foundations*, (1999), Springer · [Zbl 0909.06001](#)
- [13] Georgescu, G.; Popescu, A., Non-dual fuzzy connections, *Arch. Math. Log.*, 43, 8, 1009-1039, (2004) · [Zbl 1060.03042](#)

- [14] Hájek, P., *Metamathematics of fuzzy logic (trends in logic)*, (2001), Springer
- [15] Kauer, M.; Krupka, M., Generating complete sublattices by methods of formal concept analysis, *Int. J. Gen. Syst.*, (2017), in press
- [16] Konecny, J., Isotone fuzzy Galois connections with hedges, *Inf. Sci.*, 181, 10, 1804-1817, (2011) · [Zbl 1226.06001](#)
- [17] Konecny, J.; Krupka, M., Block relations in formal fuzzy concept analysis, *Int. J. Approx. Reason.*, 73, 27-55, (2016) · [Zbl 1352.68232](#)
- [18] Konecny, J.; Krupka, M., Complete relations on fuzzy complete lattices, *Fuzzy Sets Syst.*, (2016), in press
- [19] Konecny, J.; Medina, J.; Ojeda-Aciego, M., Multi-adjoint concept lattices with heterogeneous conjunctors and hedges, *Ann. Math. Artif. Intell.*, 72, 1-2, 73-89, (2014) · [Zbl 1322.06004](#)
- [20] Krajčí, S., Cluster based efficient generation of fuzzy concepts, *Neural Netw. World*, 13, 5, 521-530, (2003)
- [21] Krajčí, S., A generalized concept lattice, *Log. J. IGPL*, 13, 5, 543-550, (2005) · [Zbl 1088.06005](#)
- [22] Medina, J.; Ojeda-Aciego, M.; Ruiz-Calvino, J., Formal concept analysis via multi-adjoint concept lattices, *Fuzzy Sets Syst.*, 160, 2, 130-144, (2009) · [Zbl 1187.68589](#)
- [23] Pollandt, S., *Fuzzy begriffe: formale begriffsanalyse von unscharfen daten*, (1997), Springer-Verlag Berlin, Heidelberg · [Zbl 0870.06008](#)
- [24] Ward, M.; Dilworth, R. P., Residuated lattices, *Trans. Am. Math. Soc.*, 45, 335-354, (1939) · [Zbl 65.0084.01](#)
- [25] Wille, R., *Restructuring lattice theory: an approach based on hierarchies of concepts*, (Rival, I., *Ordered Sets*, Boston, (1982)), 445-470
- [26] Wille, R., Complete tolerance relations of concept lattices, (Eigenthaler, G.; et al., *Contrib. Gen. Algebra*, vol. 3, (1985), Hölder-Pichler-Tempsky Wien), 397-415
- [27] Xie, W.; Zhang, Q.; Fan, L., The Dedekind-macneille completions for fuzzy posets, *Fuzzy Sets Syst.*, 160, 16, 2292-2316, (2009) · [Zbl 1185.06003](#)
- [28] Yahia, S. B.; Jaoua, A., Discovering knowledge from fuzzy concept lattice, (Kacprzyk, J.; Kandel, A.; Last, M.; Bunke, H., *Data Mining and Computational Intelligence*, (2001), Physica-Verlag GmbH Heidelberg, Germany), 167-190
- [29] Yao, W., Quantitative domains via fuzzy sets: part I: continuity of fuzzy directed complete posets, *Fuzzy Sets Syst.*, 161, 7, 973-987, (2010) · [Zbl 1193.06007](#)

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