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Denoising and inpainting of images using TV-type energies: theoretical and computational aspects. (English. Russian original) [Zbl 1381.94011](#)

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Summary: We discuss variational approaches towards the denoising of images and towards the image inpainting problem combined with simultaneous denoising. Our techniques are based on variants of the TV-model, but in contrast to this case a complete analytical theory is available in our setting. At the same time, numerical experiments illustrate the advantages of our models in comparison with some established techniques.

MSC:

- [94A08](#) Image processing (compression, reconstruction, etc.) in information and communication theory
- [49J10](#) Existence theories for free problems in two or more independent variables
- [68U10](#) Computing methodologies for image processing

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