On static solutions of the Einstein-scalar field equations.

Summary: In this article we study self-gravitating static solutions of the Einstein-Scalar Field system in arbitrary dimensions. We discuss the existence of geodesically complete solutions depending on the form of the scalar field potential $V(\phi)$, and provide full global geometric estimates when the solutions exist. The most complete results are obtained for the physically important Klein-Gordon field and are summarised as follows. When $V(\phi) = m^2|\phi|^2$, it is proved that geodesically complete solutions have Ricci-flat spatial metric, have constant lapse and are vacuum, (that is $\phi$ is constant and equal to zero if $m \neq 0$). In particular, when the spatial dimension is three, the only such solutions are either Minkowski or a quotient thereof (no nontrivial solutions exist). When $V(\phi) = m^2|\phi|^2 + 2\Lambda$, that is, when a vacuum energy or a cosmological constant is included, it is proved that no geodesically complete solution exists when $\Lambda > 0$, whereas when $\Lambda < 0$ it is proved that no non-vacuum geodesically complete solution exists unless $m^2 < -2\Lambda/(n-1)$, ($n$ is the spatial dimension) and the spatial manifold is non-compact. The proofs are based on novel techniques in comparison geometry à la Bakry-Émery that have their own interest.

MSC: 83C15 Exact solutions to problems in general relativity and gravitational theory 53Z05 Applications of differential geometry to physics 83E15 Kaluza-Klein and other higher-dimensional theories

Keywords: static solutions; scalar fields; Klein-Gordon; Backry-Émery; Einstein-scalar field equations

Full Text: DOI arXiv

References:

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.