Brüning, E.; **Nagamachi, S.**

Parametrizations of degenerate density matrices. (English) Zbl 1380.15031

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This paper develops a new parametrization for degenerate density matrices, i.e., for positive matrices in Hilbert spaces with trace equal to 1. The problem pertains to quantum systems and physics in general and is studied here for finite dimensions. The two linked conditions for density matrices introduce dependencies for their entries and therefore a parametrization is desirable that eliminates the redundancies effectively. A new continuous form is given for such a parametrization that does not rely solely on Lie algebras but in addition also uses the theory of homogeneous spaces. Two simple low dimensional examples complete the paper.

Reviewer: Frank Uhlig (Auburn)

MSC:

15B48 Positive matrices and their generalizations; cones of matrices
81Q80 Special quantum systems, such as solvable systems
22E70 Applications of Lie groups to the sciences; explicit representations
15A18 Eigenvalues, singular values, and eigenvectors

Keywords:

density matrix; degenerate spectrum; parametrization; positive matrices; Hilbert space; quantum systems

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