

Dao Quang Khai

Well-posedness for the Navier-Stokes equations with datum in the Sobolev spaces. (English)

Zbl 1375.35311

Acta Math. Vietnam. 42, No. 3, 431-443 (2017).

Summary: In this paper, we study local well-posedness for the Navier-Stokes equations with arbitrary initial data in homogeneous Sobolev spaces $\dot{H}_p^s(\mathbb{R}^d)$ for $d \geq 2$, $p > \frac{d}{2}$, and $\frac{d}{p} - 1 \leq s < \frac{d}{2p}$. The obtained result improves the known ones for $p > d$ and $s = 0$ (see [M. Cannone, Ondelettes, paraproducts, et Navier-Stokes. Paris: Diderot (1995; Zbl 1049.35517); M. Cannone and Y. Meyer, Methods Appl. Anal. 2, No. 3, 307–319 (1995; Zbl 0842.35074)]). In the case of critical indexes $s = \frac{d}{p} - 1$, we prove global well-posedness for Navier-Stokes equations when the norm of the initial value is small enough. This result is a generalization of the one in [M. Cannone, Rev. Mat. Iberoam. 13, No. 3, 515–541 (1997; Zbl 0897.35061)] in which $p = d$ and $s = 0$.

MSC:

35Q30 Navier-Stokes equations

76D05 Navier-Stokes equations for incompressible viscous fluids

35A01 Existence problems for PDEs: global existence, local existence, non-existence

35A02 Uniqueness problems for PDEs: global uniqueness, local uniqueness, non-uniqueness

Cited in 1 Document

Keywords:

Navier-Stokes equations; existence and uniqueness of local and global mild solutions; critical Sobolev and Besov spaces

Full Text: DOI

References:

- [1] Bourgain, J; Pavlović, N, Ill-posedness of the Navier-Stokes equations in a critical space in 3D, J. Funct. Anal., 255, 2233-2247, (2008) · Zbl 1161.35037 · doi:10.1016/j.jfa.2008.07.008
- [2] Bourdaud, G.: Ce qu'il faut savoir sur les espaces de Besov Prépublication de l'Université Paris 7 (janvier 1993)
- [3] Bourdaud, G, Réalisation des espaces de Besov homogènes, Ark. Mat., 26, 41-54, (1988) · Zbl 0661.46026 · doi:10.1007/BF02386107
- [4] Cannone, M.: Ondelettes, Paraproducts et Navier-Stokes, p 191. Diderot Editeur, Paris (1995) · Zbl 1049.35517
- [5] Cannone, M, A generalization of a theorem by Kato on Navier-Stokes equations, Rev. Mat. Iberoam., 13, 515-541, (1997) · Zbl 0897.35061 · doi:10.4171/RMI/229
- [6] Cannone, M; Meyer, Y, Littlewood-Paley decomposition and the Navier-Stokes equations, Methods Appl. Anal., 2, 307-319, (1995) · Zbl 0842.35074
- [7] Chemin, JM, Remarques sur l'existence globale pour le système de Navier-Stokes incompressible, SIAM J. Math. Anal., 23, 20-28, (1992) · Zbl 0762.35063 · doi:10.1137/0523002
- [8] Fabes, E; Jones, B; Riviere, NN, The initial value problem for the Navier-Stokes equations with data in l_p , Arch. Rat. Mech. Anal., 45, 222-240, (1972) · Zbl 0254.35097 · doi:10.1007/BF00281533
- [9] Fujita, H; Kato, T, On the Navier-Stokes initial value problem I, Arch. Rat. Mech. Anal., 16, 269-315, (1964) · Zbl 0126.42301 · doi:10.1007/BF00276188
- [10] Giga, Y, Solutions of semilinear parabolic equations in l_p and regularity of weak solutions of the Navier-Stokes system, J. Differ. Equ., 62, 186-212, (1986) · Zbl 0577.35058 · doi:10.1016/0022-0396(86)90096-3
- [11] Giga, Y; Miyakawa, T, Solutions in l_r of the Navier-Stokes initial value problem, Arch. Rat. Mech. Anal., 89, 267-281, (1985) · Zbl 0587.35078 · doi:10.1007/BF00276875
- [12] Khai, DQ; Tri, NM, Solutions in mixed-norm Sobolev-Lorentz spaces to the initial value problem for the Navier-Stokes equations, J. Math. Anal. Appl., 417, 819-833, (2014) · Zbl 1308.35166 · doi:10.1016/j.jmaa.2014.03.068
- [13] Khai, DQ; Tri, NM, Well-posedness for the Navier-Stokes equations with datum in Sobolev-Fourier-Lorentz spaces, J. Math. Anal. Appl., 437, 754-781, (2016) · Zbl 1339.35214 · doi:10.1016/j.jmaa.2016.01.015
- [14] Khai, DQ; Tri, NM, On the Hausdorff dimension of the singular set in time for weak solutions to the nonstationary Navier-Stokes equation on torus, Vietnam J. Math., 43, 283-295, (2015) · Zbl 1326.35242 · doi:10.1007/s10013-014-0117-8

- [15] Khai, D.Q.; Tri, N.M., On the initial value problem for the Navier-Stokes equations with the initial datum in critical Sobolev and Besov spaces, *J. Math. Sci. Univ. Tokyo*, 23, 499-528, (2016) · [Zbl 1342.35220](#)
- [16] Khai, D.Q., Tri, N.M.: Well-posedness for the Navier-Stokes equations with data in homogeneous Sobolev-Lorentz spaces. preprint, arXiv:1601.01742 · [Zbl 1358.35093](#)
- [17] Khai, D.Q., Tri, N.M.: The existence and decay rates of strong solutions for Navier-Stokes Equations in Bessel-potential spaces. preprint, arXiv:1603.01896 · [Zbl 1326.35242](#)
- [18] Khai, D.Q., Tri, N.M.: The existence and space-time decay rates of strong solutions to Navier-Stokes equations in weighed $L^p(\mathbb{R}^n)$ spaces. preprint, arXiv:1601.01723 · [Zbl 1161.35037](#)
- [19] Kato, T; Fujita, H, On the non-stationary Navier-Stokes system, *Rend. Sem. Mat. Univ. Padova*, 32, 243-260, (1962) · [Zbl 0114.05002](#)
- [20] Kato, T.: Strong L^p solutions of the Navier-Stokes equations in \mathbb{R}^m with applications to weak solutions. *Math. Zeit.* 187, 471-480 (1984) · [Zbl 0545.35073](#)
- [21] Kato, T, Strong solutions of the Navier-Stokes equations in Morrey spaces, *Bol. Soc. Brasil. Math.*, 22, 127-155, (1992) · [Zbl 0781.35052](#) · [doi:10.1007/BF01232939](#)
- [22] Koch, H; Tataru, D, Well-posedness for the Navier-Stokes equations, *Adv. Math.*, 157, 22-35, (2001) · [Zbl 0972.35084](#) · [doi:10.1006/aima.2000.1937](#)
- [23] Lemarie-Rieusset, P.G.: *Recent Developments in the Navier-Stokes Problem*. Chapman and Hall/CRC Research Notes in Mathematics, vol. 431, p 395. Chapman and Hall/CRC, Boca Raton, FL (2002) · [Zbl 1034.35093](#)
- [24] Taylor, ME, Analysis on Morrey spaces and applications to Navier-Stokes equations and other evolution equations, *Comm. P. D. E*, 17, 1407-1456, (1992) · [Zbl 0771.35047](#) · [doi:10.1080/03605309208820892](#)
- [25] Weissler, FB, The Navier-Stokes initial value problem in L^p , *Arch. Rat. Mech. Anal.*, 74, 219-230, (1981) · [Zbl 0454.35072](#) · [doi:10.1007/BF00280539](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.