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On the convergence of the iterates of the “fast iterative shrinkage/thresholding algorithm”.

(English) [Zbl 1371.65047](#)

J. Optim. Theory Appl. 166, No. 3, 968-982 (2015).

Summary: We discuss here the convergence of the iterates of the “Fast Iterative Shrinkage/Thresholding Algorithm”, which is an algorithm proposed by *A. Beck* and *M. Teboulle* [*SIAM J. Imaging Sci.* 2, No. 1, 183–202 (2009; [Zbl 1175.94009](#))] for minimizing the sum of two convex, lower-semicontinuous, and proper functions (defined in a Euclidean or Hilbert space), such that one is differentiable with Lipschitz gradient, and the proximity operator of the second is easy to compute. It builds a sequence of iterates for which the objective is controlled, up to a (nearly optimal) constant, by the inverse of the square of the iteration number. However, the convergence of the iterates themselves is not known. We show here that with a small modification, we can ensure the same upper bound for the decay of the energy, as well as the convergence of the iterates to a minimizer.

MSC:

[65J15](#) Numerical solutions to equations with nonlinear operators

[65Y20](#) Complexity and performance of numerical algorithms

[90C25](#) Convex programming

Cited in **1** Review
Cited in **99** Documents

Keywords:

optimization; first-order schemes; convergence; forward backward splitting; inertial algorithms

Software:

UNLocBoX

Full Text: [DOI](#)

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