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Multiplicative controllability for semilinear reaction-diffusion equations with finitely many changes of sign. (English. French summary) Zbl 1370.93140

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Summary: We study the global approximate controllability properties of a one-dimensional semilinear reaction-diffusion equation governed via the coefficient of the reaction term. It is assumed that both the initial and target states admit no more than finitely many changes of sign. Our goal is to show that any target state, with as many changes of sign in the same order as the given initial data, can be approximately reached in the $L^2(0,1)$-norm at some time $T > 0$. Our method employs shifting the points of sign change by making use of a finite sequence of initial-value pure diffusion problems.

MSC:

93C20 Control/observation systems governed by partial differential equations
35K55 Nonlinear parabolic equations
35K10 Second-order parabolic equations
35K57 Reaction-diffusion equations
35K58 Semilinear parabolic equations

Keywords:

semilinear parabolic equations; reaction-diffusion equations; bilinear controls; approximate controllability

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