Non-finitely generated relatively hyperbolic groups and Floyd quasiconvexity. (English)
Zbl 1364.20032

Summary: We regard a relatively hyperbolic group as a group acting non-trivially by homeomorphisms on a compactum T discontinuously on the set of distinct triples and cocompactly on the set of distinct pairs of points of T.

In the first part of the paper we prove that such a group G admits a graph of groups decomposition given by a star graph whose central vertex group is finitely generated relatively hyperbolic with respect to the edge groups, and the other vertex groups are stabilizers of non-equivalent parabolic points. It follows from this result that every relatively hyperbolic group is relatively finitely generated with respect to the parabolic subgroups. Another corollary is that the definition of the relative hyperbolicity which we are using is equivalent to those of Bowditch and Osin (taken with respect to finitely many peripheral subgroups) and they are all equivalent to the existence of the above star graph of groups decomposition.

The second part of the paper uses the method of the first part. Considering the induced action of G on the space of distinct pairs of T we construct a connected graph on which G acts properly and cofinitely on edges. Equipping the graph with Floyd metrics we prove that the quasigeodesics in this metric are close somewhere to the geodesics in the word metric. This allows us to prove that the parabolic subgroups of G are quasiconvex with respect to the Floyd metrics. As a corollary we prove that the preimage of a parabolic point by the Floyd map is the Floyd boundary of its stabilizer.

MSC:

20F67	Hyperbolic groups and nonpositively curved groups
20F65	Geometric group theory
57M07	Topological methods in group theory
22D05	General properties and structure of locally compact groups
20F05	Generators, relations, and presentations of groups

Cited in **8** Documents

Keywords:

relatively hyperbolic group; relatively finitely generated group; Floyd quasiconvexity

Full Text: DOI arXiv

References:

[1] A. Beardon and B. Maskit, Limit points of Kleinian groups and finite sided funda- mental polyhedra. Acta Math. 132 (1974), 1-12. · Zbl 0277.30017 · doi:10.1007/BF02392106

[2] B. H. Bowditch, Relatively hyperbolic groups. Internat. J. Algebra Comput. 22 (2012), no. 3, article id. 1250016, 66 pp. · Zbl 1259.20052 · doi:10.1142/S0218196712500166

[3] B. H. Bowditch, Convergence groups and configuration spaces. In J. Cossey, Ch. F. Miller III, W. D. Neumann, and M. Shapiro (eds.), Geometric group theory down under. (Canberra, July 14-19, 1996.) Walter de Gruyter & Co., Berlin, 1999, 23-54. · Zbl 0952.20032

[4] B. H. Bowditch, A topological characterisation of hyperbolic groups. J. Amer. Math. Soc. 11 (1998), no. 3, 643-667. · Zbl 0906.20022 · doi:10.1090/S0894-0347-98-00264-1

[5] B. H. Bowditch, A short proof that a subquadratic isoperimetric inequality implies a linear one. Michigan Math. J. 42 (1995), no. 1, 103-107. · Zbl 0835.53051 · doi:10.1307/mmj/1029005156

[6] M. Bridson and A. Haefliger, Metric spaces of non-positive curvature. Grundlehren der Mathematischen Wissenschaften, 319. Springer, Berlin etc., 1999. · Zbl 0988.53001

[7] N. Bourbaki, Éléments de mathématique. Topologie générale. Chapitres 1 à 4. Her- mann, Paris, 1971. · Zbl 0249.54001

[8] D. Burago, Y. Burago, and S. Ivanov, A course in metric geometry. Graduate Stud- ies in Mathematics, 33. American Mathematical Society, Providence, R.I., 2001. · Zbl 0981.51016

[9] F. Dahmani, Les groupes relativement hyperboliques et leurs bords. Ph.D. Thesis. Université Louis Pasteur (Strasbourg I), Strasbourg, 2003.

[10] C. Drutu and M. Sapir, Tree-graded spaces and asymptotic cones of groups. With an appendix by D. Osin and M. Sapir.

Topology 44 (2005), no. 5, 959-1058. · Zbl 1101.20025 · doi:10.1016/j.top.2005.03.003

[11] B. Farb, Relatively hyperbolic groups. Geom. Funct. Anal. 8 (1998), no. 5, 810-840. · Zbl 0985.20027 · doi:10.1007/s000390050075

[12] W. J. Floyd, Group completions and limit sets of Kleinian groups. Invent. Math. 57 (1980), no. 3, 205-218. · Zbl 0428.20022 · doi:10.1007/BF01418926

[13] E. M. Freden, Properties of convergence groups and spaces. Conform. Geom. Dyn. 1 (1997), 13-23 (electronic). · Zbl 0983.57029 · doi:10.1090/S1088-4173-97-00011-8

[14] H. Furstenberg, Poisson boundaries and envelopes of discrete groups. Bull. Amer. Math. Soc. 73 (1967), 350-356. · Zbl 0184.33105 · doi:10.1090/S0002-9904-1967-11748-8

[15] V. Gerasimov, Expansive convergence groups are relatively hyperbolic. Geom. Funct. Anal. 19 (2009), no. 1, 137-169. · Zbl 1226.20037 · doi:10.1007/s00039-009-0718-7

[16] V. Gerasimov, Floyd maps for relatively hyperbolic groups. Geom. Funct. Anal. 22 (2012), no. 5, 1361-1399. · Zbl 1276.20050 · doi:10.1007/s00039-012-0175-6

[17] V. Gerasimov and L. Potyagailo, Quasi-isometric maps and Floyd boundaries of rel- atively hyperbolic groups. J. Eur. Math. Soc. (JEMS) 15 (2013), no. 6, 2115-2137. · Zbl 1292.20047 · doi:10.4171/JEMS/417

[18] V. Gerasimov and L. Potyagailo, Quasiconvexity in the relatively hyperbolic groups. Preprint 2011. · Zbl 1361.20031

[19] F. W. Gehring and G. J. Martin, Discrete quasiconformal groups I. Proc. London Math. Soc. (3) 55 (1987), no. 2, 331-358. · Zbl 0628.30027 · doi:10.1093/plms/s3-55$_2$.331

[20] M. Gromov, Hyperbolic groups. In S. M. Gersten (ed.), Essays in group theory. Springer, New York, 1987, 75-263. · Zbl 0634.20015