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Optimal Sharpe ratio in continuous-time markets with and without a risk-free asset. (English) Zbl 1361.90047

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Summary: In this paper, we investigate a continuous-time mean-variance portfolio selection model with only risky assets and its optimal Sharpe ratio in a new way. We obtain closed-form expressions for the efficient investment strategy, the efficient frontier and the optimal Sharpe ratio. Using these results, we further prove that (i) the efficient frontier with only risky assets is significantly different from the one with inclusion of a risk-free asset and (ii) inclusion of a risk-free asset strictly enhances the optimal Sharpe ratio. Also, we offer an explicit expression for the enhancement of the optimal Sharpe ratio. Finally, we test our theory results using an empirical analysis based on real data of Chinese equity market. Out-of-sample analyses shed light on advantages of our theoretical results established.

MSC:

91G10 Portfolio theory
93E20 Optimal stochastic control

Keywords:

continuous-time mean-variance model; efficient investment strategy; efficient frontier; Sharpe ratio; Hamilton-Jacobi-Bellman equation

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