Deraux, Martin

A 1-parameter family of spherical CR uniformizations of the figure eight knot complement.

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The author studies some properties of the structures modelled on the boundary of a symmetric 4-space X of negative curvature. The visual boundary $\partial_\infty X$ is a 3-dimensional sphere if $X = H^4_{\mathbb{R}}$ of $H^2_{\mathbb{C}}$. The author is interested in a special kind of spherical CR-structures, namely spherical CR-uniformizations. Such CR-structures are characterized by the fact that the developing map of the structure is a diffeomorphism onto its image, which is an open set in S^3. The holonomy group is a discrete subgroup $\Gamma \subset \mathrm{PU}(2,1)$ and the image of the developing map is the domain of discontinuity Ω_Γ of Γ. The quotient $\Gamma\backslash\Omega_\Gamma$ is the manifold at infinity of Γ. Let M denote the figure eight knot complement. The author shows that M admits a 1-parameter family of pairwise nonconjugate spherical CR-uniformizations. He follows closely the results from [*M. Deraux* and *E. Falbel*, Geom. Topol. 19, No. 1, 237–293 (2015; Zbl 1335.32028)]. The obtained unique spherical CR uniformization with unipotent boundary is called the boundary unipotent uniformization of M and the corresponding holonomy representation is denoted by ρ.

One of the first results gives an explicit construction of twist-parabolic deformations:

Theorem 1.1. There is a continuous 1-parameter family of irreducible representations $\rho_t : \pi_1(M) \to \mathrm{PU}(2,1)$ such that ρ_t (for each t) maps peripheral subgroups of M onto a cyclic group generated by a single parabolic element with eigenvalues e^{it}, e^{it}, e^{-2it}.

The author uses an elementary construction, which is close to the parametrization of the character variety of the figure eight knot group into $\mathrm{PSL}_2(\mathbb{C})$, to obtain

Theorem 1.2. There exists a $\delta > 0$ such that for $|t| < \delta$, ρ_t is the holonomy of a spherical CR uniformization of the figure eight knot complement.

In the proof of this result, the author studies the Ford domain for the image of ρ_0, and shows that it is generic enough for its combinatorics to be preserved under small deformations of ρ_0. It follows that the Ford domain exhibits an explicit horotube structure for the group. The author describes the Ford domain for the holonomy group of the unipotent uniformization of M, and studies in detail the generic character of the intersection of its sides, along faces of all dimensions. The deformations which the author considers preserve the conjugacy classes of the elliptic elements and they do not affect the nongeneric character of the fundamental domains of the considered points.

Proposition 1.3. The image of ρ_t is a triangle group. More specifically, for all t, one has $\rho_t(g_2)^4 = \rho_t(g_1g_2)^3 = \rho_t(g_2g_1g_2)^3 = \mathrm{id}$.

The following topics are considered: the real hyperbolic Ford domain, basic complex hyperbolic geometry, the Siegel domain and the Heisenberg group, Ford domains and the Poincaré polyhedron theorem, a boundary parabolic family of representations, fixed points of elliptic elements, combinatorics of the Ford domain in the unipotent case, side pairings, ridge cycles, topology of the manifold at infinity, stability of the combinatorics, stability of the side pairing.

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