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**Nonlinear flexural analysis of laminated composite panel under hygro-thermo-mechanical loading – a micromechanical approach.** (English) [Zbl 1359.74339](#)  
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