Khai, D. Q.; **Tri, N. M.**

Well-posedness for the Navier-Stokes equations with data in homogeneous Sobolev-Lorentz spaces. (English) Zbl 1358.35093

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This paper is concerning the solutions of Navier-Stokes equations in some specific L^∞ space when the initial conditions belong to a certain Sobolev-Lorentz space, more general than the cases studied before. A more general result is obtained, compared with [*M. Cannone* et al., Ondelettes, paraproduits, et Navier-Stokes. Paris: Diderot (1995; Zbl 1049.35517); Methods Appl. Anal. 2, No. 3, 307–319 (1995; Zbl 0842.35074)]. A much weaker condition on the initial data is imposed. The existence of the mild solution is given, when the norm of initial data in a specific Besov space is small enough.

Reviewer: Gelu Paşa (Bucureşti)

MSC:

35Q30 Navier-Stokes equations
76D05 Navier-Stokes equations for incompressible viscous fluids
76N10 Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics

Keywords:

Navier-Stokes equations; mild solutions; existence and uniqueness; homogeneous Sobolev-Lorentz spaces

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