Lou, Yifei; **Yin, Penghang**; **Xin, Jack**
Point source super-resolution via non-convex L_1 based methods. (English) Zbl 1354.65125
J. Sci. Comput. 68, No. 3, 1082-1100 (2016).

In the super-resolution problem, one has to recover sparse signals $x \in \mathbb{R}^N$ consisting only of separated peaks from frequency measurements up to a low frequency cutoff f_c. Then $\lambda_c = 1/f_c$ is called Rayleigh length of x. If the peak separation Δ of x fulfills $\Delta \geq 2\lambda_c N$, then such sparse signal x can be recovered by ℓ_1 minimization (see [*E. J. Candès* and *C. Fernandez-Granda*, Commun. Pure Appl. Math. 67, No. 6, 906–956 (2014; Zbl 1350.94011)]).

In this paper, the authors analyze the so-called minimum separation factor MSF $= \Delta/(\lambda_c N)$. Thus MSF ≥ 2 guarantees the exact recovery by ℓ_1 minimization. Numerical tests show that ℓ_1 minimization often fails when MSF < 1. In this case, the authors investigate the recovery of $x = (x_j)_{j=1}^N$ by non-convex ℓ_1 based minimization with the regularization term $R(x) = \|x\|_1 - \|x\|_2$ or

$$R(x) = \sum_{j=1}^{N} \min\{|x_j|,\, \alpha\}$$

with some $\alpha > 0$. The non-convex ℓ_1 based minimization problem is solved via a difference of convex algorithms such that local minimizers are obtained. The authors show several properties of the local minimizers. Numerical experiments for signals and images are examined.

Reviewer: Manfred Tasche (Rostock)

MSC:

65K05 Numerical mathematical programming methods
94A12 Signal theory (characterization, reconstruction, filtering, etc.)
65J22 Numerical solution to inverse problems in abstract spaces
65T40 Numerical methods for trigonometric approximation and interpolation
90C26 Nonconvex programming, global optimization

Keywords:

super-resolution; Rayleigh length; minimum separation factor; non-convex ℓ_1 based minimization; reconstruction of sparse signals; ℓ_1 minimization; numerical test; regularization; numerical experiment

Software:

BLOOMP

Full Text: DOI

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