Alves, Claudianor O.; Figueiredo, Giovany M.; Yang, Minbo

Existence of solutions for a nonlinear Choquard equation with potential vanishing at infinity. (English) Zbl 1354.35029

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Summary: We study the following nonlinear Choquard equation:

$$-\Delta u + V(x)u = \left(\frac{1}{|x|^{\mu}} * F(u)\right) f(u) \qquad \text{in } \mathbb{R}^N,$$

where $0 < \mu < N$, $N \geq 3$, V is a continuous real function and F is the primitive function of f. Under some suitable assumptions on the potential V, which include the case $V(\infty) = 0$, that is, $V(x) \to 0$ as $|x| \to +\infty$, we prove the existence of a nontrivial solution for the above equation by the penalization method.

MSC:

35J20 Variational methods for second-order elliptic equations
35J60 Nonlinear elliptic equations
35A15 Variational methods applied to PDEs

Keywords:

nonlinear Choquard equation; nonlocal nonlinearities; vanishing potential; variational methods

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