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A discontinuous Galerkin method with plane waves for sound-absorbing materials. (English)

Zbl 1352.74145
Int. J. Numer. Methods Eng. 104, No. 12, 1115-1138 (2015).

Summary: Poro-elastic materials are commonly used for passive control of noise and vibration and are key to reducing noise emissions in many engineering applications, including the aerospace, automotive and energy industries. More efficient computational models are required to further optimise the use of such materials. In this paper, we present a discontinuous Galerkin method (DGM) with plane waves for poro-elastic materials using the Biot theory solved in the frequency domain. This approach offers significant gains in computational efficiency and is simple to implement (costly numerical quadratures of highly oscillatory integrals are not needed). It is shown that the Biot equations can be easily cast as a set of conservation equations suitable for the formulation of the wave-based DGM. A key contribution is a general formulation of boundary conditions as well as coupling conditions between different propagation media. This is particularly important when modelling porous materials as they are generally coupled with other media, such as the surround fluid or an elastic structure. The validation of the method is described first for a simple wave propagating through a porous material, and then for the scattering of an acoustic wave by a porous cylinder. The accuracy, conditioning and computational cost of the method are assessed, and comparison with the standard finite element method is included. It is found that the benefits of the wave-based DGM are fully realised for the Biot equations and that the numerical model is able to accurately capture both the oscillations and the rapid attenuation of the waves in the porous material.

MSC:

74J15 Surface waves in solid mechanics
74S05 Finite element methods applied to problems in solid mechanics
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
74F10 Fluid-solid interactions (including aero- and hydro-elasticity, porosity, etc.)

Keywords:

porous material; Biot theory; discontinuous Galerkin method; plane wave

Full Text: DOI

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