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**The stability of stars of simplicial hybrid equilibrium finite elements for solid mechanics.**  
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**Summary:** In this paper, we define the spurious kinematic modes of hybrid equilibrium 2D and 3D simplicial elements of general degree and present the results of studies on the stability of star patches of such elements. The approach used in these studies is based on first establishing the kinematic properties of a pair of elements that share an interface. This approach is introduced by its application to star patches of hybrid equilibrium triangular plate elements for modelling membrane and plate bending problems and then generalised to 3D continua. It is then shown how the existence of spurious kinematic modes depends on the topological and geometrical properties of a patch, as well as on the degree of the polynomial approximation functions of stress and displacement.

**MSC:**

- [74A05](#) Kinematics of deformation
- [74S05](#) Finite element methods applied to problems in solid mechanics
- [65N30](#) Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs

**Keywords:**

[solid mechanics](#); [finite elements](#); [hybrid equilibrium](#); [spurious kinematic modes](#)

**Software:**

[DLMF](#)

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