Ventura, G.; **Benvenuti, E.**

Equivalent polynomials for quadrature in Heaviside function enriched elements. (English)

Zbl 1352.65559

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Summary: One of the advantages of partition-of-unity FEMs, like the extended FEM, is the ability of modeling discontinuities independent of the mesh structure. The enrichment of the element functional space with discontinuous or non-differentiable functions requires, when the element stiffness is computed, partitioning into subdomains for quadrature. However, the arbitrary intersection between the base mesh and the discontinuity plane generates quadrature subdomains of complex shape. This is particularly true in three-dimensional problems, where quite sophisticate methodologies have been presented in the literature for the element stiffness evaluation.

The present work addresses the problem of Heaviside function enrichments and is based on the replacement of the discontinuous enrichment function with the limit of an equivalent polynomial defined on the entire element domain. This allows for the use of standard Gaussian quadrature in the elements crossed by the discontinuity. The work redefines conceptually the first version of the equivalent polynomial methodology introduced in [*G. Ventura*, Int. J. Numer. Methods Eng. 66, No. 5, 761–795 (2006; Zbl 1110.74858)], allowing a much broader applicability. As a consequence, equivalent polynomials can be computed for all continuum element families in one, two, and three dimensions.

MSC:

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs

65D30 Numerical integration

Cited in **12** Documents

Software:

Cubpack++; XFEM; e$_f$*loat*

Full Text: DOI

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