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Positivity-preserving flux difference splitting schemes. (English) Zbl 1349.76519

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Summary: A positivity-preserving variant of the Roe flux difference splitting method is here proposed. Positivity-preservation is attained by modifying the Roe scheme such that the coefficients of the discretization equation become positive, with a coefficient considered positive if all its eigenvalues are positive and if its eigenvectors correspond to those of the flux Jacobian. Because the modification does not alter the wave speeds at the interface, the appealing attributes of the Roe flux difference splitting schemes are retained, such as high-resolution capture of discontinuous waves, low amount of artificial dissipation within viscous layers, and ease of convergence to steady-state. The proposed flux function is advantaged over previous positivity-preserving variants of the Roe method by being written in general matrix form and hence by being readily deployable to arbitrary systems of conservation laws. The stencils are extended to second-order accuracy through a newly-derived positivity-preserving total-variation-diminishing limiting process that is applied to the characteristic variables and that yields positive coefficients. Also derived is a positivity-preserving restriction on the time step for flux difference splitting schemes that is shown to depart significantly from the CFL condition in regions with high property gradients.

MSC:

76M20 Finite difference methods applied to problems in fluid mechanics
65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs

Cited in **1** Document

Keywords:

positivity preservation; rule of the positive coefficients; flux difference splitting (FDS); Roe solver; monotonicity preservation; total variation diminishing (TVD); centered TVD limiters; Yee-Roe scheme; interface averaging

Software:

HLLE

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