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**Accuracy analysis of mimetic finite volume operators on geodesic grids and a consistent alternative.** (English) [Zbl 1349.76376](#)  
*J. Comput. Phys.* 310, 127-160 (2016).

Summary: Many newly developed climate, weather and ocean global models are based on quasi-uniform spherical polygonal grids, aiming for high resolution and better scalability. *J. Thuburn* et al. [ibid. 228, No. 22, 8321–8335 (2009; [Zbl 1173.86304](#))] and *T. D. Ringler* et al. [ibid. 229, No. 9, 3065–3090 (2010; [Zbl 1307.76054](#))] developed a C staggered finite volume/difference method for arbitrary polygonal spherical grids suitable for these next generation dynamical cores. This method has many desirable mimetic properties and became popular, being adopted in some recent models, in spite of being known to possess low order of accuracy. In this work, we show that, for the nonlinear shallow water equations on non-uniform grids, the method has potentially 3 main sources of inconsistencies (local truncation errors not converging to zero as the grid is refined): (i) the divergence term of the continuity equation, (ii) the perpendicular velocity and (iii) the kinetic energy terms of the vector invariant form of the momentum equations. Although some of these inconsistencies have not impacted the convergence on some standard shallow water test cases up until now, they may constitute a potential problem for high resolution 3D models. Based on our analysis, we propose modifications for the method that will make it first order accurate in the maximum norm. It preserves many of the mimetic properties, albeit having non-steady geostrophic modes on the f-sphere. Experimental results show that the resulting model is a more accurate alternative to the existing formulations and should provide means of having a consistent, computationally cheap and scalable atmospheric or ocean model on C staggered Voronoi grids.

**MSC:**

- [76M12](#) Finite volume methods applied to problems in fluid mechanics
- [86-08](#) Computational methods for problems pertaining to geophysics
- [65M08](#) Finite volume methods for initial value and initial-boundary value problems involving PDEs
- [76B15](#) Water waves, gravity waves; dispersion and scattering, nonlinear interaction
- [86A05](#) Hydrology, hydrography, oceanography
- [86A10](#) Meteorology and atmospheric physics

Cited in 4 Documents

**Keywords:**

shallow water model; finite volume; staggered C grid; spherical Voronoi grid; mimetic discretization

**Software:**

MPAS-Ocean; chammp; DYNAMICO

**Full Text:** [DOI](#)

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