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High order ADER schemes for a unified first order hyperbolic formulation of continuum mechanics: viscous heat-conducting fluids and elastic solids. (English) [Zbl 1349.76324](#)
J. Comput. Phys. 314, 824-862 (2016).

Summary: This paper is concerned with the numerical solution of the *unified* first order hyperbolic formulation of continuum mechanics recently proposed by the second and the third author [Contin. Mech. Thermodyn. 28, No. 1–2, 85–104 (2016; [Zbl 1348.76046](#))], further denoted as *HPR model*. In that framework, the viscous stresses are computed from the so-called *distortion tensor* A , which is one of the primary state variables in the proposed first order system. A very important key feature of the HPR model is its ability to describe *at the same time* the behavior of inviscid and viscous compressible Newtonian and non-Newtonian *fluids* with heat conduction, as well as the behavior of elastic and viscoplastic *solids*. Actually, the model treats viscous and inviscid fluids as generalized visco-plastic solids. This is achieved via a stiff source term that accounts for strain relaxation in the evolution equations of A . Also heat conduction is included via a first order hyperbolic system for the thermal impulse, from which the heat flux is computed. The governing PDE system is hyperbolic and fully consistent with the first and the second principle of thermodynamics. It is also fundamentally *different* from first order Maxwell-Cattaneo-type relaxation models based on extended irreversible thermodynamics. The HPR model represents therefore a *novel* and *unified* description of continuum mechanics, which applies at the same time to *fluid mechanics* and *solid mechanics*. In this paper, the direct connection between the HPR model and the classical hyperbolic-parabolic Navier-Stokes-Fourier theory is established for the first time via a formal asymptotic analysis in the stiff relaxation limit. From a numerical point of view, the governing partial differential equations are very challenging, since they form a large nonlinear hyperbolic PDE system that includes stiff source terms and non-conservative products. We apply the successful family of one-step ADER-WENO finite volume (FV) and ADER discontinuous Galerkin (DG) finite element schemes to the HPR model in the stiff relaxation limit, and compare the numerical results with exact or numerical reference solutions obtained for the Euler and Navier-Stokes equations. Numerical convergence results are also provided. To show the universality of the HPR model, the paper is rounded-off with an application to wave propagation in elastic solids, for which one only needs to switch off the strain relaxation source term in the governing PDE system. We provide various examples showing that for the purpose of *flow visualization*, the distortion tensor A seems to be particularly useful.

MSC:

- 76M12 Finite volume methods applied to problems in fluid mechanics
- 74S10 Finite volume methods applied to problems in solid mechanics
- 76M10 Finite element methods applied to problems in fluid mechanics
- 74S05 Finite element methods applied to problems in solid mechanics
- 65M08 Finite volume methods for initial value and initial-boundary value problems involving PDEs
- 65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs

Cited in **42** Documents

Keywords:

ADER-WENO finite volume schemes; arbitrary high-order discontinuous Galerkin schemes; path-conservative methods and stiff source terms; unified first order hyperbolic formulation of nonlinear continuum mechanics; fluid mechanics and solid mechanics; viscous compressible fluids and elastic solids

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