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Homogenization methods for multi-phase elastic composites with non-elliptical reinforcements: comparisons and benchmarks. (English) [Zbl 1348.74292](#)

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Summary: The purpose of this work is comparing three strategies for dealing with inhomogeneities of non-elliptical shape in the context of homogenization methods. First, classical mean-field methods and two relatively new approaches, IDD and ESCS, are used in combination with analytical expressions for the Eshelby tensor based on its irreducible decomposition. The second strategy to be investigated is the Mori-Tanaka method in combination with the replacement tensor approach, which uses numerical models of dilute inhomogeneities embedded in large matrix regions. The third approach consists of the direct Finite Element discretization of microstructures. The elasticity tensors and directional Young's moduli are first studied for arrangements of aligned inhomogeneities of three different shapes and of combinations of these shapes. Subsequently the three modeling strategies are applied to a real microstructure. Comparisons are not only carried out with respect to phase volume fractions, but also with respect to the contrast in the elastic phase properties. All calculations are restricted to plane strain conditions and to isotropic material behavior.

MSC:

[74Q15](#) Effective constitutive equations in solid mechanics

[74A60](#) Micromechanical theories

[74E30](#) Composite and mixture properties

[74G10](#) Analytic approximation of solutions (perturbation methods, asymptotic methods, series, etc.) of equilibrium problems in solid mechanics

[74N15](#) Analysis of microstructure in solids

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