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Hamiltonian cycles in linear-convex supergrid graphs. (English) Zbl 1348.05117

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Summary: A supergrid graph is a finite induced subgraph of the infinite graph associated with the two-dimensional supergrid. The supergrid graphs contain grid graphs and triangular grid graphs as subgraphs. The Hamiltonian cycle problem for grid and triangular grid graphs was known to be NP-complete. Recently, we have proved the Hamiltonian cycle problem on supergrid graphs to be NP-complete. The Hamiltonian cycle problem on supergrid graphs can be applied to control the stitching trace of computerized sewing machines. In this paper, we will study the Hamiltonian cycle property of linear-convex supergrid graphs which form a subclass of supergrid graphs. A connected graph is called k-connected if there are k vertex-disjoint paths between every pair of vertices, and is called locally connected if the neighbors of each vertex in it form a connected subgraph. In this paper, we first show that any 2-connected, linear-convex supergrid graph is locally connected. We then give constructive proofs to show that any 2-connected, linear-convex supergrid graph contains a Hamiltonian cycle. Based on the constructive proofs, we finally present a linear-time algorithm to construct a Hamiltonian cycle of a 2-connected, linear-convex supergrid graph.

MSC:

05C45 Eulerian and Hamiltonian graphs
05C38 Paths and cycles

Keywords:

Hamiltonian cycle; locally connected; linear-convex supergrid graph; supergrid graph; grid graph; triangular grid graph; computer sewing machine

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