Matano, H.; **Poláčik, P.**

Dynamics of nonnegative solutions of one-dimensional reaction-diffusion equations with localized initial data. Part I: A general quasiconvergence theorem and its consequences.

(English) Zbl 1345.35052
Commun. Partial Differ. Equations 41, No. 5, 785-811 (2016).

Summary: We consider the Cauchy problem

$$u_1 = u_{xx} + f(u),\ x \in \mathbb{R}, t > 0,$$

$$u(x,0) = u_0(x),\ x \in \mathbb{R},$$

where f is a locally Lipschitz function on $\mathbb{R}$ with $f(0) = 0$, and u_0 is a nonnegative function in $C_0(\mathbb{R})$, the space of continuous functions with limits at $\pm$ equal to 0. Assuming that the solution u is bounded, we study its large-time behavior from several points of view. One of our main results is a general quasiconvergence theorem saying that all limit profiles of $u(\cdot,t)$ in $L^\infty_{loc}(\mathbb{R})$ are steady states. We also prove convergence results under additional conditions on u_0. In the bistable case, we characterize the solutions on the threshold between decay to zero and propagation to a positive steady state and show that the threshold is sharp for each increasing family of initial data in $C_0(\mathbb{R})$.

MSC:

35K15 Initial value problems for second-order parabolic equations
35B40 Asymptotic behavior of solutions to PDEs

Cited in **1** Review
Cited in **14** Documents

Keywords:

convergence; generalized omega-limit set; localized initial data; parabolic equations on $\mathbb{R}$; quasiconvergence; threshold solutions

Full Text: DOI

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