Goldreich, Oded; **Zuckerman, David**

Another proof that $\mathcal{BPP} \subseteq \mathcal{PH}$ (and more). (English) Zbl 1343.68085

Goldreich, Oded (ed.), Studies in complexity and cryptography. Miscellanea on the interplay between randomness and computation. In collaboration with Lidor Avigad, Mihir Bellare, Zvika Brakerski, Shafi Goldwasser, Shai Halevi, Tali Kaufman, Leonid Levin, Noam Nisan, Dana Ron, Madhu Sudan, Luca Trevisan, Salil Vadhan, Avi Wigderson, David Zuckerman. Berlin: Springer (ISBN 978-3-642-22669-4/pbk). Lecture Notes in Computer Science 6650, 40-53 (2011).

Summary: We provide another proof of the Sipser-Lautemann Theorem by which $\mathcal{BPP} \subseteq \mathcal{MA}$ ($\subseteq \mathcal{PH}$). The current proof is based on strong results regarding the amplification of $\mathcal{BPP}$, due to *D. Zuckerman* [Algorithmica 16, No. 4–5, 367–391 (1996; Zbl 0857.68121)]. Given these results, the current proof is even simpler than previous ones. Furthermore, extending the proof leads to two results regarding $\mathcal{MA}$: $\mathcal{MA} \subseteq \mathcal{ZPP}^{\mathcal{NP}}$ (which seems to be new), and that two-sided error $\mathcal{MA}$ equals $\mathcal{MA}$. Finally, we survey the known facts regarding the fragment of the polynomial-time hierarchy that contains $\mathcal{MA}$.

For the entire collection see [Zbl 1220.68005].

MSC:

68Q15 Complexity classes (hierarchies, relations among complexity classes, etc.)

Keywords:

BPP; polynomial-time hierarchy; interactive proof systems (AM and MA); randomness-efficient error reduction (amplification)

Full Text: DOI

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