Let $G$ be a finite group and $\Lambda(G)$ be the set of irreducible complex representations of $G$. Associated to a fixed finite-dimensional representation $V$, are the numbers $m^\lambda_k$, which are the multiplicity of $\lambda \in \Lambda(G)$ in $V^\otimes k$. $A = (a_{\lambda \mu})$, the adjacency matrix of the representation graph $R_V(G)$, and the Bratteli diagram which is the infinite graph with vertices labeled by the elements of $\Lambda(G)$ on level $k$, keeping tracks of finite steps walks on $R_V(G)$. The Poincaré series for $\lambda \in \Lambda(G)$ is defined by

$$m^\lambda(t) = \sum_{k=0}^{\infty} m^\lambda_{tk}.\$$

The main result of the paper under review is as follows: suppose that $G$ acts faithfully on $V$, and $V^* \cong V$ as $G$-representations. Let $M_\mu$ be the matrix $I - tA$ with the column indexed by $\mu$ replaced by $(1 \ 0 \ 0 \ldots \ 0)^T$. Then

$$m^\mu(t) = \frac{\det M_\mu}{\det(I - tA)},$$

In the special case where $G$ is a finite subgroup of $SU(2)$, and $V = \mathbb{C}^2$ is its natural representation, then

$$m^0(t) = \frac{\det(I - \hat{A})}{\det(I - tA)},$$

where $0 \in \Lambda(G)$ is the trivial representation, and $\hat{A}$ is the adjacency matrix of the finite Dynkin diagram obtained by removing the affine node. In the continuation, the paper under review finds further explicit formulas for $m^0(t)$ and express the results in terms of Chebyshev polynomials, and the Bratteli diagrams.

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