Hindes, Wade

The Vojta conjecture implies Galois rigidity in dynamical families. (English) Zbl 1338.14026

Proc. Am. Math. Soc. 144, No. 5, 1931-1938 (2016).

The author proves that the Hall-Lang conjecture (or Vojta's conjecture) implies Galois rigidity in a dynamical arboreal setting. More specifically, the author considers the one-parameter family $\phi_a(x) = (x-\gamma(a))^2 + c(a)$ of quadratic polynomials, where $\gamma, c \in \mathbb{Z}[t]$ and $a \in \mathbb{Z}$. Let $G_n(\phi_a)$ be the Galois group of the splitting field $K_n(\phi_a)$ of the n-th iterate ϕ_a^n of ϕ_a; this acts on a binary rooted tree T_n. Taking inverse limit, we obtain an arboreal representation of $G_\infty(\phi_a)$, serving as an analog of the ℓ-adic Galois representations for elliptic curves. The main result is the following:

Theorem 1. If ϕ_t is not isotrivial and $\phi_t(\gamma(t)) \cdot \phi_t^2(\gamma(t)) \neq 0$, then Hall-Lang (or Vojta) conjecture shows that there exist an integer n and an effectively computable finite set F such that for all $a \in \mathbb{Z}\backslash F$, $G_n(\phi_a) \cong \mathrm{Aut}(T_n)$ implies $G_\infty(\phi_a) \cong \mathrm{Aut}(T_\infty)$. Moreover, there exists a uniform bound on $[\mathrm{Aut}(T_\infty) : G_\infty(\phi_a)]$ for $a \in \mathbb{Z}\backslash F$ for which all iterates of ϕ_a are irreducible.

As a corollary, when $G_\infty(\phi_a)$ is maximal, the density of primes dividing the orbit of $b \in \mathbb{Z}$ under ϕ_a is shown to be 0. The general strategy of the proof is similar to [*W. Hindes*, Acta Arith. 169, No. 1, 1–27 (2015; Zbl 1330.14032)]: the bounds on integral/rational points on genus 1 or 2 curve (coming from Hall-Lang or Vojta) forces a square-free primitive prime divisor in the critical orbit, but if $K_n(\phi_a)/K_{n-1}(\phi_a)$ is not maximal, $\phi_a^n(\gamma(a))$ must be a square in $K_{n-1}(\phi_a)$, so such a primitive divisor ramifies, contradicting a discriminant formula.

Reviewer: Yu Yasufuku (Tokyo)

MSC:

14G05 Rational points
37P45 Families and moduli spaces in arithmetic and non-Archimedean dynamical systems
11R32 Galois theory
37P05 Arithmetic and non-Archimedean dynamical systems involving polynomial and rational maps
11J97 Number-theoretic analogues of methods in Nevanlinna theory (work of Vojta et al.)
37P55 Arithmetic dynamics on general algebraic varieties

Keywords:

rational points on curves; arithmetic dynamics; Galois theory; arboreal representation; Vojta conjecture

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