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Mixed convolved action principles in linear continuum dynamics. (English) Zbl 1336.74013

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Summary: The paper begins with an overview of several of the classical integral formulations of elastodynamics, which highlights the natural appearance of temporal convolutions in the reciprocal theorem for such problems. This leads first to the formulation of a principle of virtual convolved action, as an extension of the principle of virtual work to dynamical problems. Then, to overcome the key shortcomings of Hamilton's principle, the concept of mixed convolved action is developed for linear dynamical problems within the context of continuum solid mechanics. This new approach is broadly applicable to both reversible and irreversible phenomena without the need for special treatments, such as the artificial definition of Rayleigh dissipation functionals. The focus here is on linear elastic and viscoelastic media, which in the latter case is represented by classical Kelvin-Voigt and Maxwell models. Remarkably, for each problem type, the stationarity of the mixed convolved action provides not only the governing partial differential equations, but also the specified boundary and initial conditions, as its Euler-Lagrange equations. Thus, the entire initial/boundary value problem definition is encapsulated in a scalar mixed convolved action functional written in terms of displacements and stress impulses. The resulting formulations possess an elegant structure that provides a versatile framework for the development of novel computational methods, involving finite element representations in both space and time. We present perhaps the simplest approach by employing linear three-node triangular elements for two-dimensional analysis, along with linear shape functions over the temporal domain. Numerical examples are included to verify the formulation and to explore concepts of stress wave attenuation.

MSC:

74D05 Linear constitutive equations for materials with memory

74B05 Classical linear elasticity

Software:

FEAPpv

Full Text: DOI

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