Rauhut, Holger; Ward, Rachel

In practice, one has often to interpolate functions which are smooth as well as sparse in some sense. In this excellent paper, the authors merge classical smoothness-based interpolation methods with modern sparsity constraints and nonlinear reconstruction methods.

For a bounded domain $D$, let $\psi_j : D \to \mathbb{C}$ ($j \in \Lambda$) be orthonormal functions with finite index set $\Lambda$, $|\Lambda| = N$. For given sampling points $t_\ell \in D$ ($\ell = 1, \ldots, m$) and $f = \sum_{j \in \Lambda} x_j \psi_j$, let $y = (f(t_\ell))_{\ell=1}^m$ and let $A$ be the sampling matrix with the entries $A_{\ell,j} = \psi_j(t_\ell)$ ($\ell = 1, \ldots, m; j \in \Lambda$). For interpolation, the authors consider the function $f^\# = \sum_{j \in \Lambda} x^\#_j \psi_j$, whose coefficient vector $x^\#$ is the solution of the weighted $\ell_1$ minimization problem

$$\min \|z\|_{\omega,1} \quad \text{subject to} \quad \|Az - y\|_2 \leq \eta$$

with the weighted $\ell_1$ norm $\|z\|_{\omega,1} = \sum_{j \in \Lambda} \omega_j |z_j|$ and convenient weights $\omega_j \geq 1$.

Using the new concepts of weighted null space property and weighted restricted isometry property of the sampling matrix $A$, the authors prove general interpolation theorems. Corresponding error estimates of $f - f^\#$ in $L_\infty$ resp. $L_2$ norm are given. In several examples and numerical tests, this theory is applied to spherical harmonic interpolation and tensorized polynomial interpolation (with Chebyshev resp. Legendre polynomials).

Reviewer: Manfred Tasche (Rostock)

MSC:
41A05 Interpolation in approximation theory
65D05 Numerical interpolation
94A20 Sampling theory in information and communication theory

Keywords:
interpolation; weighted $\ell_1$-minimization; error estimates; smooth and sparse functions; compressive sensing; bounded orthonormal system; weighted restricted isometry property; weighted null space property; sampling matrix

Full Text: DOI

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