

**Meyer, Daniel**

**Invariant Peano curves of expanding Thurston maps.** (English) Zbl 1333.37043

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Summary: We consider *Thurston maps*, i.e., branched covering maps  $f : S^2 \rightarrow S^2$  that are *post-critically finite*. In addition, we assume that  $f$  is *expanding* in a suitable sense. It is shown that each sufficiently high iterate  $F = f^n$  of  $f$  is *semi-conjugate* to  $z^d : S^1 \rightarrow S^1$ , where  $d = \deg F$ . More precisely, for such an  $F$  we construct a *Peano curve*  $\gamma : S^1 \rightarrow S^2$  (onto), such that  $F \circ \gamma(z) = \gamma(z^d)$  (for all  $z \in S^1$ ).

**MSC:**

**37F20** Combinatorics and topology in relation with holomorphic dynamical systems

**37D20** Uniformly hyperbolic systems (expanding, Anosov, Axiom A, etc.)

**37F10** Dynamics of complex polynomials, rational maps, entire and meromorphic functions; Fatou and Julia sets

**57M12** Low-dimensional topology of special (e.g., branched) coverings

**57M50** General geometric structures on low-dimensional manifolds

**28A80** Fractals

Cited in **1** Review  
Cited in **6** Documents

**Full Text:** [DOI](#) [arXiv](#)

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