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On the topology of random complexes built over stationary point processes. (English)

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Summary: There has been considerable recent interest, primarily motivated by problems in applied algebraic topology, in the homology of random simplicial complexes. We consider the scenario in which the vertices of the simplices are the points of a random point process in $\mathbb{R}^d$, and the edges and faces are determined according to some deterministic rule, typically leading to Čech and Vietoris-Rips complexes. In particular, we obtain results about homology, as measured via the growth of Betti numbers, when the vertices are the points of a general stationary point process. This significantly extends earlier results in which the points were either i.i.d. observations or the points of a Poisson process. In dealing with general point processes, in which the points exhibit dependence such as attraction or repulsion, we find phenomena quantitatively different from those observed in the i.i.d. and Poisson cases. From the point of view of topological data analysis, our results seriously impact considerations of model (non)robustness for statistical inference. Our proofs rely on analysis of subgraph and component counts of stationary point processes, which are of independent interest in stochastic geometry.

MSC:

60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)
60D05 Geometric probability and stochastic geometry
60G10 Stationary stochastic processes
05E45 Combinatorial aspects of simplicial complexes
55U10 Simplicial sets and complexes in algebraic topology
05C10 Planar graphs; geometric and topological aspects of graph theory
58K05 Critical points of functions and mappings on manifolds

Keywords:

stationary point processes; random geometric complexes; Čech complexes; Vietoris-Rips complexes; component counts; Betti numbers; Morse critical points

Software:

javaPlex

Full Text: DOI Euclid arXiv

References:

[1] Babson, E., Hoffman, C. and Kahle, M. (2011). The fundamental group of random 2-complexes. J. Amer. Math. Soc. 24 1-28. · Zbl 1270.20042 · doi:10.1090/S0894-0347-2010-00677-7

[2] Björner, A. (1995). Topological methods. In Handbook of Combinatorics , Vol. 1, 2 1819-1872. Elsevier, Amsterdam. · Zbl 0851.52016

[3] Błaszczyszyn, B. and Yogeshwaran, D. (2015). Clustering, percolation and comparison of point processes. In Stochastic Geometry , Spatial Statistics and Random Fields : Models and Algorithms (V. Schmidt, ed.). Lecture Notes in Mathematics 2120 . Springer, Cham. · Zbl 1328.60122

[4] Błaszczyszyn, B. and Yogeshwaran, D. (2013). Clustering and percolation of point processes. Electron. J. Probab. 18 no. 72, 20. · Zbl 1291.60099 · doi:10.1214/EJP.v18-2468

[5] Błaszczyszyn, B. and Yogeshwaran, D. (2014). On comparison of clustering properties of point processes. Adv. in Appl. Probab. 46 1-20. · Zbl 1295.60059 · doi:10.1239/aap/1396360100

[6] Bobrowski, O. and Adler, R. J. (2014). Distance functions, critical points, and topology for some random complexes. Homology , Homotopy Appl. 16 311-344. · Zbl 1323.60026 · doi:10.4310/HHA.2014.v16.n2.a18

[7] Bobrowski, O. and Mukherjee, S. (2015). The topology of probability distributions on manifolds. Probab. Theory Related Fields 161 651-686. · Zbl 1316.60020 · doi:10.1007/s00440-014-0556-x

[8] Bobrowski, O., Mukherjee, S. and Taylor, J. (2014). Topological consistency via kernel estimation. Preprint. Available at .

arXiv:1407.5272 · Zbl 1395.62073

[9] Bubenik, P., Carlsson, G., Kim, P. T. and Luo, Z.-M. (2010). Statistical topology via Morse theory persistence and nonparametric estimation. In Algebraic Methods in Statistics and Probability II. Contemp. Math. 516 75-92. Amer. Math. Soc., Providence, RI. · Zbl 1196.62041 · doi:10.1090/conm/516/10167

[10] Burton, R. and Waymire, E. (1985). Scaling limits for associated random measures. Ann. Probab. 13 1267-1278. · Zbl 0579.60039 · doi:10.1214/aop/1176992810

[11] Carlsson, G. (2009). Topology and data. Bull. Amer. Math. Soc. (N.S.) 46 255-308. · Zbl 1172.62002 · doi:10.1090/S0273-0979-09-01249-X

[12] Carlsson, G. (2014). Topological pattern recognition for point cloud data. Acta Numer. 23 289-368. · Zbl 1398.68615 · doi:10.1017/S0962492914000051

[13] Chazal, F., Cohen-Steiner, D. and Mérigot, Q. (2011). Geometric inference for probability measures. Found. Comput. Math. 11 733-751. · Zbl 1230.62074 · doi:10.1007/s10208-011-9098-0

[14] Cohen, D., Costa, A., Farber, M. and Kappeler, T. (2011). Topology of random 2-complexes. Discrete Comput. Geom. 47 No. 1. · Zbl 1237.55009

[15] Edelsbrunner, H. and Harer, J. L. (2010). Computational Topology : An Introduction . Amer. Math. Soc., Providence, RI. · Zbl 1193.55001

[16] Forman, R. (2002). A user's guide to discrete Morse theory. Sém. Lothar. Combin. 48 Art. B48c, 35. · Zbl 1048.57015

[17] Gershkovich, V. and Rubinstein, H. (1997). Morse theory for Min-type functions. Asian J. Math. 1 696-715. · Zbl 0921.58006

[18] Ghosh, S. (2012). Determinantal processes and completeness of random exponentials: The critical case. Preprint. Available at . arXiv:1211.2435

[19] Ghrist, R. (2008). Barcodes: The persistent topology of data. Bull. Amer. Math. Soc. (N.S.) 45 61-75. · Zbl 1391.55005 · doi:10.1090/S0273-0979-07-01191-3

[20] Hough, J. B., Krishnapur, M., Peres, Y. and Virág, B. (2009). Zeros of Gaussian Analytic Functions and Determinantal Point Processes. University Lecture Series 51 . Amer. Math. Soc., Providence, RI. · Zbl 1190.60038

[21] Kahle, M. (2009). Topology of random clique complexes. Discrete Math. 309 1658-1671. · Zbl 1215.05163 · doi:10.1016/j.disc.2008.02.037

[22] Kahle, M. (2011). Random geometric complexes. Discrete Comput. Geom. 45 553-573. · Zbl 1219.05175 · doi:10.1007/s00454-010-9319-3

[23] Kahle, M. (2014). Topology of random simplicial complexes: A survey. In Algebraic Topology : Applications and New Directions. Contemp. Math. 620 (U. Tillmann, S. Galatius and D. Sinha, eds.). Amer. Math. Soc., Providence, RI. · Zbl 1333.05324

[24] Kahle, M. and Meckes, E. (2013). Limit theorems for Betti numbers of random simplicial complexes. Homology , Homotopy Appl. 15 343-374. · Zbl 1268.05180 · doi:10.4310/HHA.2013.v15.n1.a17

[25] Kallenberg, O. (1983). Random Measures , 3rd ed. Akademie-Verlag, Berlin. · Zbl 0345.60032

[26] Lavancier, F., Møller, J. and Rubak, E. (2012). Determinantal point process models and statistical inference. Preprint. Available at . arXiv:1205.4818

[27] Linial, N. and Meshulam, R. (2006). Homological connectivity of random 2-complexes. Combinatorica 26 475-487. · Zbl 1121.55013 · doi:10.1007/s00493-006-0027-9

[28] Meester, R. and Roy, R. (1996). Continuum Percolation. Cambridge Tracts in Mathematics 119 . Cambridge Univ. Press, Cambridge. · Zbl 0858.60092 · doi:10.1017/CBO9780511895357

[29] Meshulam, R. and Wallach, N. (2009). Homological connectivity of random \(k\)-dimensional complexes. Random Structures Algorithms 34 408-417. · Zbl 1177.55011 · doi:10.1002/rsa.20238

[30] Mileyko, Y., Mukherjee, S. and Harer, J. (2011). Probability measures on the space of persistence diagrams. Inverse Problems 27 124007, 22. · Zbl 1247.68310 · doi:10.1088/0266-5611/27/12/124007

[31] Miyoshi, N. and Shirai, T. (2013). Cellular networks with \(\alpha\)-Ginibre configurated base stations. Available at . · Zbl 1344.60050 · doi:10.1007/978-4-431-54907-$9_1$6

[32] Nazarov, F. and Sodin, M. (2012). Correlation functions for random complex zeroes: Strong clustering and local universality. Comm. Math. Phys. 310 75-98. · Zbl 1238.60059 · doi:10.1007/s00220-011-1397-4

[33] Pemantle, R. (2000). Towards a theory of negative dependence. J. Math. Phys. 41 1371-1390. · Zbl 1052.62518 · doi:10.1063/1.533200

[34] Pemantle, R. and Peres, Y. (2014). Concentration of Lipschitz functionals of determinantal and other strong Rayleigh measures. Combin. Probab. Comput. 23 140-160. · Zbl 1326.60032 · doi:10.1017/S0963548313000345

[35] Penrose, M. (2003). Random Geometric Graphs. Oxford Studies in Probability 5 . Oxford Univ. Press, Oxford. · Zbl 1029.60007

[36] Schneider, R. and Weil, W. (2008). Stochastic and Integral Geometry . Springer, Berlin. · Zbl 1175.60003 · doi:10.1007/978-3-540-78859-1

[37] Shirai, T. and Takahashi, Y. (2003). Random point fields associated with certain Fredholm determinants. I. Fermion, Poisson and boson point processes. J. Funct. Anal. 205 414-463. · Zbl 1051.60052 · doi:10.1016/S0022-1236(03)00171-X

[38] Stoyan, D., Kendall, W. and Mecke, J. (1995). Stochastic Geometry and Its Applications . Wiley, Chichester. · Zbl 0838.60002

[39] Tausz, A. and Vejdemo-Johansson, M. (2011). JavaPlex: A research software package for persistent (co) homology. Available at . · Zbl 1402.65186

[40] Turner, K., Mileyko, Y., Mukherjee, S. and Harer, J. (2014). Fréchet means for distributions of persistence diagrams. Discrete Comput. Geom. 52 44-70. · Zbl 1296.68182 · doi:10.1007/s00454-014-9604-7

[41] Yogeshwaran, D., Subag, E. and Adler, R. J. (2014). Random geometric complexes in the thermodynamic regime. Preprint. Available at . arXiv:1403.1164 · Zbl 1366.60033

[42] Zomorodian, A. J. (2009). Topology for Computing. Cambridge Monographs on Applied and Computational Mathematics 16 . Cambridge Univ. Press, Cambridge. · Zbl 1182.68002