Cabrer, Leonardo Manuel; **Mundici, Daniele**
Interval MV-algebras and generalizations. (English) Zbl 1326.06011
Int. J. Approx. Reasoning 55, No. 8, 1623-1642 (2014).

In this paper, the set of all intervals of an MV-algebra A is equipped with the following operations and constants: $\neg I = \{\neg x \mid x \in I\}$; $I \oplus J = \{x \oplus y \mid x \in I,\ y \in J\}$; $\Delta(I) = [\min I, \min I]$; $\nabla I = [\max I, \max I]$; $0 = [0, 0]$; $1 = [1, 1]$; and $i = A$. The resulting algebraic structure is called $I(A)$, the interval algebra of A; moreover the interval algebra construction is functorial. The models of certain finitely many equational properties of $I(A)$ are called IMV-algebras.

It is shown that every IMV-algebra is isomorphic to $I(B)$ for some MV-algebra B. The category of IMV-algebras is shown to be equivalent to the category of MV-algebras, and its free objects are characterized. Then Łukasiewicz interval logic is defined as the deductive system whose inference rules are semantic consequence relations between IMV-terms. The tautology and consequence problems for this logic are coNP-complete.

Finally a vast generalization of the interval algebra construction is performed: instead of MV-algebras, a large class of quasivarieties of partially ordered algebras is considered, where the operations are monotone or antimonotone in each variable. It results that the interval algebra functor is an equivalence for many quasivarieties, and necessary and sufficient conditions are given for this to happen.

Reviewer: Giacomo Lenzi (Fisciano)

MSC:

06D35 MV-algebras
03G25 Other algebras related to logic

Keywords:

approximated reasoning; interval MV-algebras; intervals as truth-values; categorical equivalences; interval logic; triangularizations; IMV-algebras

Full Text: DOI arXiv

References:

[1] Baczyński, M.; Jayaram, B., (S,N)- and R-implications: a state-of-the-art survey, Fuzzy Sets Syst., 159, 1836-1859, (2008) · Zbl 1175.03013

[2] Callejas-Bedregal, R.; Bedregal, B. C., Acióly-Scott interval categories, Electron. Notes Theor. Comput. Sci., 95, 169-187, (2004)

[3] Bedregal, B. C.; Callejas-Bedregal, R.; Santos, H. S., Bounded lattice t-norms as an interval category, (Leivant, D.; de Queiroz, R., WoLLIC, Lecture Notes in Computer Science, vol. 4576, (2007)), 26-37 · Zbl 1213.03034

[4] Bedregal, B. C.; Nunes Santiago, R. H., Interval representations, łukasiewicz implicators and smets-magrez axioms, Inf. Sci., 221, 192-200, (2013) · Zbl 1293.03013

[5] Birkhoff, G., Lattice-ordered groups, Ann. Math. (2), 43, 2, 298-331, (1942) · Zbl 0060.05808

[6] Boicescu, V.; Filipoiu, A.; Georgescu, G.; Rudeanu, S., Lukasiewicz-moisil algebras, (1991), North-Holland Amsterdam · Zbl 0726.06007

[7] Bou, F.; Esteva, F.; Godo, L., On the minimum many-valued modal logic over a finite residuated lattice, J. Log. Comput., 21, 739-790, (2011) · Zbl 1252.03040

[8] Burris, S.; Sankappanavar, H. P., A course in universal algebra, Graduate Texts in Mathematics, vol. 78, (1981), Springer Berlin · Zbl 0478.08001

[9] Chagrov, A.; Zakharyaschev, M., Modal logic, Oxford Logic Guides, vol. 35, (1997), Oxford University Press · Zbl 0871.03007

[10] Chajda, I.; Kühr, J., A note on interval MV-algebras, Math. Slovaca, 56, 1, 47-52, (2006) · Zbl 1164.06010

[11] Chang, C. C., A new proof of the completeness of the łukasiewicz axioms, Trans. Am. Math. Soc., 93, 74-80, (1959) · Zbl 0093.01104

[12] Cignoli, R., Coproducts in the categories of Kleene and three-valued łukasiewicz algebras, Stud. Log., 38, 237-245, (1979) ·

Zbl 0432.03038

[13] Cignoli, R.; D'Ottaviano, I. M.L.; Mundici, D., Algebraic foundations of many-valued reasoning, Trends in Logic, vol. 7, (2000), Kluwer Academic Publishers Dordrecht · Zbl 0937.06009

[14] (Cornelis, C.; Deschrijver, G.; Nachtegael, M.; Schockaert, S.; Shi, Yun, 35 Years of Fuzzy Set Theory, Studies in Fuzziness and Soft Computing, vol. 261, (2010), Springer Berlin)

[15] Li, Dechao; Li, Yongming, Algebraic structures of interval-valued fuzzy (S,N)-implications, Int. J. Approx. Reason., 53, 892-900, (2012) · Zbl 1251.03032

[16] Diego, A., Sur LES algébres de Hilbert, Colléction de Logique Mathèmatique Serie A, vol. 21, (1966), Gauthier-Villars Paris · Zbl 0144.00105

[17] Di Nola, A.; Grigolia, R., On monadic MV-algebras, Ann. Pure Appl. Log., 128, 125-139, (2004) · Zbl 1052.06010

[18] Dubois, D., On degrees of truth, partial ignorance and contradiction, (Magdalena, L.; etal., Proceedings of IPMU'08, Torremolinos, Spain, (2008)), 31-38

[19] Entemann, C. W., A fuzzy logic with interval truth values, Fuzzy Sets Syst., 113, 2, 161-183, (2000) · Zbl 0953.03029

[20] Esteva, F.; Garcia-Calvés, P.; Godo, L., Enriched interval bilattices: an approach to deal with uncertainty and imprecision, Int. J. Uncertain. Fuzziness Knowl.-Based Syst., 1, 37-54, (1994) · Zbl 1232.03013

[21] Galatos, N.; Jipsen, P.; Kowalski, T.; Ono, H., Residuated lattices. an algebraic glimpse at substructural logics, Stud. Logic Found. Math., vol. 151, (2007), Elsevier · Zbl 1171.03001

[22] Garey, M. R.; Johnson, D. S., Computers and intractability: A guide to the theory of NP-completeness, (1979), W.H. Freeman New York · Zbl 0411.68039

[23] Gehrke, M.; Walker, C.; Walker, E., Some comments on interval valued fuzzy sets, Int. J. Intell. Syst., 11, 751-759, (1996) · Zbl 0865.04006

[24] Hájek, P., Metamathematics of fuzzy logic, trends in logic, vol. 4, (1998), Kluwer Academic Publishers Dordrecht

[25] Harlenderová, M.; Rachunek, J., Modal operators on MV-algebras, Math. Bohem., 131, 1, 39-48, (2006) · Zbl 1112.06014

[26] Kenevan, J. R.; Neapolitan, R. E., A model theoretic approach to propositional fuzzy logic using beth tableaux, (Zadeh, L.; Kacprzyk, J., Fuzzy Logic for the Management of Uncertainty, (1992), Wiley New York, NY), 141-157

[27] McKenzie, R.; McNulty, G.; Taylor, W., Algebras, lattices, varieties, vol. I, (1987), Wadsworth & Brooks/Cole Belmont, CA

[28] McLane, S., Categories for the working Mathematician, Graduate Texts in Mathematics, vol. 5, (1998), Springer Berlin

[29] Mendel, J. M., Uncertain rule-based fuzzy logic systems, (2001), Prentice Hall, PTR Upper Saddle River, NJ · Zbl 0978.03019

[30] Monteiro, A., Construction des algébres de łukasiewicz trivalentes dans LES algébres de Boole monadiques I, Math. Jpn., 12, 1, 1-23, (1967) · Zbl 0165.30903

[31] Moore, R. E.; Baker Kearfott, R.; Cloud, M. J., Introduction to interval analysis, (2009), SIAM Philadelphia · Zbl 1168.65002

[32] Mukaidono, M., Algebraic structures of interval truth values in fuzzy logic, (Proceedings of the Sixth IEEE International Conference on Fuzzy Systems, vol. 2, (1997)), 699-706

[33] Mundici, D., Interpretation of AF $C^\ast$-algebras in łukasiewicz sentential calculus, J. Funct. Anal., 65, 15-63, (1986) · Zbl 0597.46059

[34] Mundici, D., Bookmaking over infinite-valued events, Int. J. Approx. Reason., 43, 223-240, (2006) · Zbl 1123.03011

[35] Mundici, D., Interpretation of de Finetti coherence criterion in łukasiewicz logic, Ann. Pure Appl. Log., 161, 235-245, (2009) · Zbl 1180.03029

[36] Mundici, D., A compact $[0, 1]$-valued first-order łukasiewicz logic with identity on Hilbert space, J. Log. Comput., 21, 3, 509-525, (2011) · Zbl 1220.03010

[37] Mundici, D., Advanced łukasiewicz calculus and MV-algebras, Trends in Logic, vol. 35, (2011), Springer NY · Zbl 1235.03002

[38] Mundici, D., The differential semantics of łukasiewicz syntactic consequence, (Montagna, F., Petr Hájek on Mathematical Fuzzy Logic, Outstanding Contributions to Logic, vol. 6, (2014), Springer International Publishing Switzerland), Chapter 7 · Zbl 1386.03022

[39] D. Pigozzi, Partially ordered varieties and quasivarieties, preprint, http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.142.3923.

[40] Rose, A.; Rosser, J. B., Fragments of many-valued statement calculi, Trans. Am. Math. Soc., 87, 1-53, (1958) · Zbl 0085.24303

[41] Smets, P.; Magrez, P., Implication in fuzzy logic, Int. J. Approx. Reason., 1, 327-347, (1987) · Zbl 0643.03018

[42] Van Gasse, B.; Cornelis, G.; Deschrijver, G.; Kerre, E. E., On the properties of a generalized class of t-norms in interval-valued fuzzy logics, New Math. Nat. Comput., 2, 29-42, (2006) · Zbl 1102.03031

[43] Van Gasse, B.; Cornelis, G.; Deschrijver, G.; Kerre, E. E., Triangle algebras: a formal logic approach to interval-valued residuated lattices, Fuzzy Sets Syst., 159, 1042-1060, (2008) · Zbl 1174.03028

[44] Weber, S., Uncertainty measures — problems concerning additivity, Fuzzy Sets Syst., 160, 371-383, (2009) · Zbl 1180.28012

[45] Zadeh, L., The concept of a linguistic variable and its application to approximate reasoning, I, Inf. Sci., 8, 199-249, (1975) · Zbl 0397.68071